There Must Be a Rational Explanation
Adding and Subtracting Rational Expressions

Problem Set
Calculate the least common denominator (LCD) for each sum and difference. Describe any restriction(s) for the value of $x$.

1. $\frac{x}{3} + \frac{x + 1}{15}$
   
   The LCD is $3(5)$, or $15$.
   
   No restrictions for $x$
   
   $\frac{x}{3} + \frac{x + 1}{3(5)}$

2. $\frac{7x}{12} + \frac{x - 2}{6} - \frac{x^2}{3}$

3. $\frac{x + 1}{x} - \frac{x - 1}{x^2 + x}$

4. $\frac{3}{2x} + \frac{x^2 + 1}{4x^2 + 8}$

5. $\frac{3x + 4}{x} - \frac{5}{6x} + \frac{9}{2x}$

6. $\frac{x}{x^2 - 1} - \frac{x - 3}{x - 1}$

7. $\frac{3x}{x - 2} + \frac{x}{2x + 4} + \frac{5}{x}$

8. $\frac{x - 3}{2x^2 + 7x + 6} - \frac{x}{2x + 3}$
Calculate each sum and difference. Simplify the answer when possible.

9. \[ \frac{x}{2} + \frac{7x}{6} \]

\[
\frac{x}{2} + \frac{7x}{6} = \frac{x(3)}{2(3)} + \frac{7x}{6}
\]
\[
= \frac{3x}{6} + \frac{7x}{6}
\]
\[
= \frac{10x}{6}
\]
\[
= \frac{5x}{3}
\]

10. \[ \frac{x + 2}{4} - \frac{z}{10} \]
11. $\frac{-2x}{5} - \frac{y}{10} + z$

12. $\frac{x - 3}{15} - \frac{x - 3}{10}$
13. \( \frac{2x}{7} - \frac{y}{2} + \frac{x + 1}{3} \)

14. \( \frac{x - 1}{4} + \frac{x + 2}{2} - \frac{x^2}{8} \)
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15. \( \frac{2x}{5} + \frac{2x^2 - 1}{10} - \frac{4x + 1}{6} \)

16. \( \frac{x + 4}{2} - \frac{x + 5}{3} + \frac{x + 6}{21} \)
Calculate each sum and difference. Describe any restriction(s) for the value of $x$ and simplify the answer when possible.

17. \( \frac{3}{x} + \frac{1}{x + 1} \)

\[
\frac{3}{x} + \frac{1}{x + 1} = \frac{3(x + 1) + 1(x)}{x(x + 1)} + \frac{1(x)}{(x + 1)(x)}
\]

\[
= \frac{3x + 3 + x}{x(x + 1)} + \frac{x}{x(x + 1)}
\]

\[
= \frac{4x + 3}{x(x + 1)}; x \neq -1, 0
\]

18. \( \frac{2}{x - 2} - \frac{5}{x + 3} \)
19. \[
\frac{x}{2x - 1} + \frac{x + 2}{x}
\]
21. \(\frac{1}{x^2 - 4} - \frac{1}{x - 2}\)

22. \(\frac{x + 3}{x - 1} + \frac{x - 4}{x + 2}\)
LESSON 10.1  Skills Practice

23. \[ \frac{x + 1}{x^2 - 16} - \frac{x}{x^2 + 7x + 12} \]

24. \[ \frac{1}{x - 4} - \frac{x}{x + 2} + \frac{x^2}{x - 1} \]
25. \( \frac{x + 1}{x^2 - 3x - 4} + \frac{x - 3}{x - 2} \)

26. \( \frac{x + 2}{2x - 2} - \frac{-2x - 1}{x^2 - 4x + 3} \)
Different Client, Same Deal
Multiplying and Dividing Rational Expressions

Problem Set
Perform the indicated operation. Simplify the answer when possible.

1. \( \frac{2}{21} \cdot \frac{3}{4} \)
   
   \( \frac{2}{21} \cdot \frac{3}{4} = \frac{1}{21} \cdot \frac{3}{2} \)
   
   \( = \frac{1}{14} \)

2. \( \frac{15}{22} \cdot \frac{8}{15} \)

3. \( \frac{27}{32} \cdot \frac{1}{8} \cdot \frac{16}{9} \)

4. \( \frac{8}{9} \div \frac{2}{3} \)

5. \( \frac{4}{21} \div \frac{12}{49} \)

6. \( \frac{1}{8} \div \frac{7}{4} + \frac{1}{14} \)
Multiply each expression. Describe any restriction(s) for the variables and simplify the answer when possible.

7. \( \frac{5x^2}{7} \cdot \frac{14}{3x} \)

\[
\frac{5x^2}{7} \cdot \frac{14}{3x} = \frac{5}{7} \cdot \frac{14^2}{3x^1}
= \frac{10x}{3}; x \neq 0
\]

8. \( \frac{2ab^2}{5c^3} \cdot \frac{15c}{4a} \)

9. \( \frac{3mn^2}{10} \cdot \frac{m^x}{8n} \cdot \frac{20}{3n^2} \)

10. \( \frac{x + 1}{x} \cdot \frac{x^2}{2x + 2} \)
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11. \[ \frac{x^2 - 4}{x + 5} \cdot \frac{x + 5}{x - 2} \]

12. \[ \frac{x^2 + 2x - 3}{x^2} \cdot \frac{x^3 + x^2}{x + 3} \]

13. \[ \frac{x^2 - 4x}{x - 2} \cdot \frac{2 - x}{x} \]

14. \[ \frac{1}{2x^2 + 3x - 2} \cdot \frac{x^2 - 2x - 8}{x - 4} \]
Determine the quotient of each expression. Describe any restriction(s) for the variables and simplify the answer when possible.

15. \( \frac{x + 3}{x - 5} \cdot \frac{1}{x^2 + 6x + 9} \cdot (x^2 - 25) \)

16. \( \frac{5x^2}{x + 4} \cdot \frac{3x^2 + 12x}{7x - 7} \cdot \frac{x^2 - 2x + 1}{3} \)

17. \( \frac{3c^2}{5ab} \div \frac{9}{2a} \)

\[
\frac{3c^2}{5ab} \div \frac{9}{2a} = \frac{3c^2}{5ab} \cdot \frac{2a}{9} \\
= \frac{3c^2}{5ab} \cdot \frac{2a}{9} \\
= \frac{3c^2}{5ab} \cdot \frac{2a}{9} \\
= \frac{3c^2}{5ab} \cdot \frac{2a}{9} \\
= \frac{2c^2}{15b} ; a \neq 0, b \neq 0
\]

18. \( \frac{4x^2y}{5z^4} \div \frac{2x}{z} \div \frac{1}{2z} \)
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19. \( \frac{x^2 + 1}{x} \div \frac{x^2 - 1}{2x} \)

20. \( \frac{x^2 + 6x - 27}{x^2} \div \frac{x^2 - 3x}{9} \)

21. \( \frac{x^2 + 6x + 8}{3x + 2} \div \frac{-x - 4}{3x^2 - x - 2} \)
22. \( \frac{x^2 - 9}{x + 3} \div (x - 3) \)

23. \( \frac{2x^2 - 2x}{x^2 + 2x + 1} \div \frac{3x - 3}{2x + 2} \)

24. \( \frac{x^2 + 4x + 3}{2x^2 - 11x + 5} \div \frac{x^2 + 3x}{2x - 1} \)
25. \[
\frac{x^2 - 121}{x^2 + x - 20} \div \frac{x^2 - 10x - 11}{x^2 - 25}
\]

26. \[
\frac{(x - 5)^2}{(x + 2)^2 (2x - 3)^2} \div \frac{(x - 5)^5}{(x + 2)(2x - 3)^2}
\]
Things Are Not Always as They Appear
Solving Rational Equations

Vocabulary
Write the term that best completes each sentence.

1. A(n) _______ is an equation containing one or more rational expressions.

2. A(n) _______ is a solution that results from the process of solving an equation; but is not a valid solution to the equation.

Problem Set
Solve each rational equation using cross multiplication. Describe any restrictions for the value of \(x\). Check your answer(s) and identify any extraneous roots should they occur.

1. \( \frac{x - 1}{x + 3} = \frac{x - 2}{x + 1} \)

   \begin{align*}
   \text{Check } x &= 5. \\
   \frac{5 - 1}{5 + 3} &= \frac{5 - 2}{5 + 1} \\
   \frac{4}{8} &= \frac{3}{6} \\
   \frac{1}{2} &= \frac{1}{2} \checkmark
   \end{align*}

   \text{Restrictions: } x \neq -3, -1

   \begin{align*}
   (x - 1)(x + 1) &= (x + 3)(x - 2) \\
   x^2 - 1 &= x^2 + x - 6 \\
   -1 &= x - 6 \\
   x &= 5
   \end{align*}
2. \( \frac{x + 2}{x - 7} = \frac{x}{x - 3} \)

3. \( \frac{2x - 1}{x + 1} = \frac{2x - 2}{x} \)
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4. \( \frac{x - 3}{x^2} = \frac{x - 3}{x^2 - 1} \)

5. \( \frac{x^2 - 1}{x - 1} = \frac{x^2 + 1}{x + 1} \)
Solve each rational equation by multiplying both sides of the equation by the least common denominator. Describe any restrictions for the value of \(x\). Check your answer(s) and identify any extraneous roots should they occur.

**6.** \(\frac{x + 5}{x - 4} = \frac{x + 4}{x - 5}\)

**7.** \(\frac{2}{x} - \frac{3}{2x} = \frac{1}{x^2}\)

Restriction: \(x \neq 0\)

\[
2x^2 \left( \frac{2}{x} - \frac{3}{2x} \right) = 2x^2 \left( \frac{1}{x^2} \right)
\]

\[
4x - 3x = 2
\]

\[
x = 2
\]

Check \(x = 2\).

\[
\frac{2}{2} - \frac{3}{2(2)} = \frac{1}{2^2}
\]

\[
1 - \frac{3}{4} = \frac{1}{4}
\]

\[
\frac{1}{4} = \frac{1}{4}
\]
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8. \( \frac{1}{x} + \frac{1}{x^2} = 2 \)

9. \( \frac{5}{2x - 4} = \frac{15}{x^2 - 4} \)
10. \( \frac{2}{x+3} + \frac{6}{x^2+3x} = \frac{1}{x} \)

11. \( \frac{2}{x^2-x} - \frac{1}{x-1} = 0 \)
12. \( \frac{x}{x + 2} + \frac{4x + 6}{2x^2 + 5x + 3} = \frac{x - 1}{2x + 4} \)
Solve each rational equation using a graphing calculator. Sketch the graph. Describe any restrictions for the value of \(x\). Check your answer(s).

13. \( \frac{x}{x+1} = \frac{3}{4} \)

Rewrite the equation so that one side equals 0, then graph \( y = \frac{x}{x+1} - \frac{3}{4} \).

The graph shows that \(x = -1\) is the location of a vertical asymptote and thus represents a restriction on the variable. The graph also shows that \(x = 3\) is a possible solution to the original rational equation.

Check \(x = 3\).

\[
\frac{3}{3+1} = \frac{3}{4}
\]

\[
\frac{3}{4} = \frac{3}{4} \quad \checkmark
\]

14. \( x + 3 = \frac{-2}{x} \)
15. \( \frac{2}{x} - \frac{1}{2} = \frac{4}{x} \)

16. \( \frac{1}{2} \cdot \frac{4}{x - 1} = \frac{x + 1}{x - 1} \)
17. \( \frac{x}{x - 2} = \frac{-1}{x} \)

18. \( \frac{x}{x - 3} = \frac{x}{x + 4} \)
Solve each rational equation without using a graphing calculator. Describe any restrictions for the value of $x$. Check your answer(s) and identify any extraneous roots should they occur.

19. \( \frac{3}{x - 1} = \frac{4}{3x + 2} \)

Restrictions: $x \neq -\frac{2}{3}, 1$

\[
(x - 1)(3x + 2) \left( \frac{3}{x - 1} \right) = (x - 1)(3x + 2) \left( \frac{4}{3x + 2} \right)
\]

\[
9x + 6 = 4x - 4
\]

\[
5x = -10
\]

\[
x = -2
\]

Check $x = -2$.

\[
\frac{3}{-2 - 1} \neq \frac{4}{3(-2) + 2}
\]

\[
\frac{3}{-3} \neq \frac{4}{-4}
\]

\[
-1 = -1 \checkmark
\]

20. \( \frac{9}{x - 3} = \frac{27}{x^2 - 3x} + \frac{6}{x} \)
21. \( \frac{x + 1}{x - 2} = \frac{3x}{x - 2} - \frac{2x + 4}{x} \)

22. \( \frac{-x}{2x + 1} = \frac{5}{x - 4} \)
23. \[ 1 + \frac{4}{x - 4} = \frac{-3}{x^2 - 16} \]

24. \[ \frac{5x}{x - 2} - 7 = \frac{10}{x - 2} \]
Get to Work, Mix It Up, Go the Distance, and Lower the Cost!
Using Rational Equations to Solve Real-World Problems

Problem Set

Write an equation to model each work scenario. Then, solve each equation.

1. Cleo can paint a room in 8 hours; while Phil can paint the same room in 6 hours. If they paint the room together, how long will it take them to paint the room?

   Working together, it will take Cleo and Phil $\frac{3}{7}$ hours to paint the room.

   Let $x$ represent the number of hours it will take to paint the room while working together.

   $\frac{x}{8} + \frac{x}{6} = 1$

   $24\left(\frac{x}{8} + \frac{x}{6}\right) = 24(1)$

   $3x + 4x = 24$

   $7x = 24$

   $x = \frac{24}{7}$ or $\frac{3}{7}$

2. Nyesha owns a lawn service company. Currently it takes her 50 hours a week to service all of her customers. To reduce the number of hours a week she needs to work, Nyesha hires Shantese to help her. While Nyesha was on vacation, Shantese was able to complete all of the work in 60 hours. If Shantese and Nyesha work together, after Nyesha returns from vacation, how long will it take them to service all their customers?
3. Using a forklift, Rico can unload a box car in 90 minutes; while Ashkii takes twice as long to complete the same task. If Rico and Ashkii work together, how long will it take them to unload a box car?

4. Yu Jie can complete a 4 foot by 6 foot quilt in 16 days; while Mufeed can complete the same task in 12 days. If they solicit Mya’s help, who can complete the task in 14 days by herself, how long will it take the three of them to complete a 4 foot by 6 foot quilt?
5. Kendall can wash 24 golf carts in a 4 hour shift. If Benny helps him they can get the job done in 2 hours. How long will it take Benny to do the job by himself?

6. Felix and Oscar own a pastry shop. Working alone Felix can decorate 8 dozen cookies in 90 minutes. Oscar, on the other hand, needs 120 minutes to decorate 8 dozen cookies. If they work together, how long does it take them to decorate 16 dozen cookies?
Write an equation to model each mixture scenario. Then, solve each equation.

7. Kaitlin knows that if she needs to add antifreeze to her car’s radiator the mixture used must contain 50% antifreeze and 50% water. How many gallons of a mixture containing 80% antifreeze must be added to a 3 gallon mixture containing 40% antifreeze to obtain the mixture Kaitlin needs?

Kaitlin needs to add 1 gallon of the 80% antifreeze mixture to the 3 gallons of the 40% antifreeze mixture to obtain a mixture containing 50% antifreeze.

Let $x$ represent the number of gallons of 80% antifreeze mixture needed.

\[
\frac{0.4(3) + 0.8x}{3 + x} = 0.5; \quad x \neq -3
\]

\[
\frac{1.2 + 0.8x}{3 + x} = 0.5
\]

\[
1.2 + 0.8x = 1.5 + 0.5x
\]

\[
0.3x = 0.3
\]

\[
x = 1
\]

8. The directions on the back of a 2 quart bottle of a 60% orange concentrate says it needs to be mixed with water to obtain a 20% orange drink. How much water should Hector add to the concentrate to obtain a drink that is 20% orange concentrate?
9. A research scientist has 10 liters of a 40% acid solution. She needs to create a solution containing 35% acid by mixing the 10 liters with a second solution containing 20% acid. How much of this second solution should she use?

10. Rosa combines 2 gallons of 2% milk and 6 gallons of 4% milk. How many additional gallons of 6% milk must she add to obtain a 5% milk mixture?
11. A jeweler has 30 ounces of an alloy consisting of 60% gold and 40% silver. How much of a second alloy containing 80% gold and 20% silver must be mixed with the first alloy to obtain an alloy containing 75% gold and 25% silver?

12. Keyon has 3 quarts of a 5% sugar solution. He wants to mix this with 2 quarts of an 8% sugar solution and $x$ quarts of a 12% sugar to solution to make a 10% sugar solution. How many quarts of the 12% sugar solution should Keyon use?
Write an equation to model each distance scenario. Then, solve each equation.

13. Kikki walked to the grocery store which was 2 miles away. Her walking rate on the way back was 0.75 of her walking rate on the way to the store because she was carrying a bag of groceries. If it took Kikki 1 hour to make the round trip, what was her walking rate on the way to the store?

Kikki’s walking rate on the way to the store was \( \frac{42}{3} \) miles per hour.

Let \( r \) represent Kikki’s walking rate on her way to the grocery store.

\[
\frac{2}{r} + \frac{2}{0.75r} = 1; \ x \neq 0
\]

\[
0.75r\left(\frac{2}{r} + \frac{2}{0.75r}\right) = 0.75r(1)
\]

\[
1.5 + 2 = 0.75r
\]

\[
3.5 = 0.75r
\]

\[
r = \frac{42}{3}
\]

14. Ricky paddles his canoe at a rate of 6 miles per hour in still water. Last weekend he canoed on Carver Creek going downstream with the current for 9 miles and then returning upstream against the current. If the trip took him 4 hours to complete, what was the speed of the current?
15. Suppose flying in calm air a robin can reach a speed of 25 kilometers per hour. Each day this robin flies from its nest to the nearest body of water 1 kilometer away. On one particular day it flew into a headwind and on its return trip the wind was at its back. If the total trip took \( \frac{1}{10} \) of an hour, what was the speed of the wind?

16. Oni walked a half a mile to her sister’s house to pick up her little brother and then walked back. The round trip took 60 minutes. If the rate at which she walked to her sister’s house was 25% faster than the rate she walked while returning home, how fast did she walk on the way home?
17. An 8-man rowing crew rows at a speed of 10 miles per hour in still water. Every morning they practice by rowing 4 miles upstream and then 4 miles downstream. If it takes them $\frac{5}{6}$ of an hour to complete the trip, what is the speed of the current?
18. Mildred runs the 100-meter dash in 15 seconds with no wind. Yesterday she ran the 100-meter dash twice. The first time she ran it in 18.75 seconds against the wind and the second time she ran it in 12.5 seconds with the wind. What was the speed of the wind?
Write an equation or inequality to model each cost scenario. Then, solve each equation or inequality.

19. Nicole purchased a power boat for $15,000 and was told by the salesman that the yearly average cost to operate the boat was approximately $900. In what year of ownership will Nicole’s average annual cost of owning the boat be $3900?

In the 5th year of ownership the average cost of owning the boat will be $3900.

Let \( x \) be the year in which Nicole’s average annual cost of owning the boat is $3900.

\[
\frac{15,000 + 900x}{x} = 3900; \quad x \neq 0
\]

\[
15,000 + 900x = 3900x
\]

\[
15,000 = 3000x
\]

\[
x = 5
\]

20. Remington purchased a new cell phone for $350 and added an annual warranty plan that cost him $35 dollars per year. In what year will Remington’s average annual cost of owning the phone be $122.50?
21. Luella always wanted a designer purse but they were too expensive to purchase. Recently she was introduced to a company that allowed individuals to rent one. The purse that Luella rented initially cost her $80 with a $15 monthly rental fee. She plans on renting a new purse until the average monthly cost of renting the purse is less than $19. When will the average monthly cost of renting the purse drop to less than $19?

22. Shopping around for a freezer, Manuit finally settled on one with a purchase price of $850. The annual cost of operating the freezer is $40 dollar per year. When Manuit's average cost of owning the freezer is less than $117, he plans to shop for a new freezer. When can Manuit shop for a new freezer?
23. Yuma purchased a flat screen TV for $700 and added an annual warranty plan that cost her $23 dollars per year. Reid purchased a similar TV for $650 and added an annual warranty plan that costs him $28 per year. In how many years will Yuma’s average annual cost of owning her TV be less than Reid’s annual cost of owning his TV?
24. Gisela is considering purchasing one of two microwave ovens. The first microwave costs $300 and comes with an annual warranty plan that costs $20 per year. The second microwave costs $350 and comes with an annual warranty plan that costs $10 per year. When will the average annual cost of owning the second microwave be less than the average annual cost of owning the first microwave?