Directions: Study the examples, work the problems, then check your answers at the end of each topic. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

**TOPIC 1: ARITHMETIC OPERATIONS**

A. Fractions:

**Simplifying fractions:**

<table>
<thead>
<tr>
<th>Example: Reduce $\frac{27}{36}$: $\frac{27}{36} = \frac{9\times3}{9\times4} = \frac{9}{9} \cdot \frac{3}{4} = 1 \cdot \frac{3}{4} = \frac{3}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Note that you must be able to find a common factor; in this case 9; in both the top and the bottom in order to reduce.)</td>
</tr>
</tbody>
</table>

Problems 1-3: Reduce:

1. $\frac{13}{52} = \frac{3}{3} \cdot \frac{3}{4} = \frac{3}{13}$
2. $\frac{26}{65} = \frac{2}{5}$
3. $\frac{3+6}{3+9} = \frac{1}{2}$

**Equivalent fractions:**

<table>
<thead>
<tr>
<th>Example: $\frac{3}{4}$ is equivalent to how many eighths?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{4} = 1 \cdot \frac{3}{2} - 2 \cdot \frac{3}{2} = 2 \cdot \frac{3}{2} = 6 \cdot \frac{3}{4} = \frac{6}{8}$</td>
</tr>
</tbody>
</table>

Problems 4-5: Complete:

4. $\frac{4}{9} = \frac{4}{72}$
5. $\frac{2}{5} = \frac{16}{20}$

How to get the **lowest common denominator** (LCD) by finding the least common multiple (LCM) of all denominators:

<table>
<thead>
<tr>
<th>Example: $\frac{5}{6}$ and $\frac{8}{15}$. First find LCM of 6 and 15:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 = 2 \cdot 3$</td>
</tr>
<tr>
<td>$15 = 3 \cdot 5$</td>
</tr>
<tr>
<td>$LCM = 2 \cdot 3 \cdot 5 = 30$, so $\frac{5}{6} = \frac{25}{30}$, and $\frac{8}{15} = \frac{16}{30}$</td>
</tr>
</tbody>
</table>

Problems 6-7: Find equivalent fractions with the LCD:

6. $\frac{2}{7}$ and $\frac{9}{2}$
7. $\frac{3}{8}$ and $\frac{7}{12}$
8. Which is larger, $\frac{5}{8}$ or $\frac{3}{4}$?
   (Hint: find the LCD fractions)

Adding, subtracting fractions: if the denominators are the same combine the numerators:

| Example: $\frac{7}{10} - \frac{1}{10} = \frac{7-1}{10} = \frac{6}{10} = \frac{3}{5}$ |

Problems 9-11: Find the sum or difference (reduce if possible):

9. $\frac{4}{7} + \frac{2}{7}$
10. $\frac{5}{6} + \frac{1}{6}$
11. $\frac{7}{8} - \frac{4}{8}$

If the denominators are different, find equivalent fractions with common denominators, then proceed as before:

<table>
<thead>
<tr>
<th>Example: $\frac{4}{5} + \frac{2}{3} = \frac{12}{15} + \frac{10}{15} = \frac{22}{15} = 1 \frac{7}{15}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example: $\frac{1}{2} - \frac{2}{3} = \frac{3}{6} - \frac{4}{6} = \frac{3-4}{6} = -\frac{1}{6}$</td>
</tr>
</tbody>
</table>

12. $\frac{3}{5} - \frac{2}{3} = \frac{9}{15} - \frac{2}{3} = \frac{11}{15} - \frac{10}{15} = \frac{1}{15}$
13. $\frac{5}{8} + \frac{1}{4} = \frac{5}{8} + \frac{2}{8} = \frac{7}{8}$

**Multiplying fractions:** multiply the top numbers, multiply the bottom numbers, reduce if possible.

| Example: $\frac{3}{4} \cdot \frac{2}{5} = \frac{3 \cdot 2}{4 \cdot 5} = \frac{6}{20} = \frac{3}{10}$ |

14. $\frac{2}{3} \cdot \frac{3}{8} = \frac{6}{24} = \frac{1}{4}$
15. $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$
17. $\left(\frac{1}{2} + \frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

**Dividing fractions:** make a compound fraction, then multiply the top and bottom (of the big fraction) by the LCD of both:

| Example: $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \cdot \frac{2}{3} = \frac{3 \cdot 2}{4 \cdot 3} = \frac{6}{12} = \frac{1}{2}$ |
| Example: $\frac{7}{3} - \frac{1}{2} \cdot \frac{6}{7} = \frac{42}{42} - \frac{42}{42} = 0$ |

18. $\frac{3}{2} \div \frac{1}{4} = \frac{3}{2} \cdot \frac{4}{1} = \frac{12}{2} = 6$
21. $\frac{2}{3} = \frac{2}{3}$

22. $\frac{22}{4}$

20. $\frac{3}{4} \div 2 = \frac{3}{4}$

B. Decimals:

Meaning of places: in 324.519, each digit position has a value ten times the place to its right. The part to the left of the point is the whole number part. Right of the point, the places have values: tenths, hundredths, etc.,

So, $324.519 = (3 \times 100) + (2 \times 10) + (4 \times 1) + (5 \times \frac{1}{10}) + (1 \times \frac{1}{100}) + (9 \times \frac{1}{1000})$.

23. Which is larger: .59 or .7?

To add or subtract decimals, like places must be combined (line up the points).

<table>
<thead>
<tr>
<th>Example: $1.23 - 1.1 = 1.13$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example: $4 + .3 = 4.3$</td>
</tr>
<tr>
<td>Example: $6.04 - (2 - 1.4) = 6.04 - .6 = 5.44$</td>
</tr>
</tbody>
</table>

24. $.54 + .78 =
25. $.36 - .63 =$

28. $1.25 + .75 = 2.00$
29. $3.3333 \div 3 = 1.1111$
30. $2.75 \times 2 = 5.5$
26. $4 - .3 + .001 - .01 + .1 = $
27. $3.54 - 1.68 = $

Multiplying decimals:

<table>
<thead>
<tr>
<th>Example</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.3 \times 0.5$</td>
<td>$0.15$</td>
</tr>
<tr>
<td>$0.3 \times 0.2$</td>
<td>$0.06$</td>
</tr>
<tr>
<td>$(0.03) \times 2$</td>
<td>$0.006$</td>
</tr>
</tbody>
</table>

28. $3.24 \times 10 = $ 30. $(0.51)^2 = $
29. $.01 \times .2 = $ 31. $5 \times .4 = $

Dividing decimals: change the problem to an equivalent whole number problem by multiplying both by the same power of ten.

<table>
<thead>
<tr>
<th>Example</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$.3 \div 0.3$</td>
<td>Multiply both by 100, to get $30 \div 3 = 10$</td>
</tr>
<tr>
<td>$0.14 \div 0.07$</td>
<td>Multiply both by 1000, get $14 \div 70 = 0.2$</td>
</tr>
</tbody>
</table>

32. $.013 \div 100 = $ 34. $\frac{340}{34} = $
33. $.053 \div .2 = $

C. Positive integer exponents and square roots of perfect squares:

<table>
<thead>
<tr>
<th>Example</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$</td>
<td>$81$</td>
</tr>
<tr>
<td>$4^2 = 4 \cdot 4$</td>
<td>$16$</td>
</tr>
</tbody>
</table>

Problems 35-44: Find the value:

35. $3^2 = $ 40. $100^2 = $
36. $-(3)^2 = $ 41. $(2.1)^2 = $
37. $-(3)^2 = $ 42. $(-1)^3 = $
38. $-3^2 = $ 43. $(\frac{2}{3})^3 = $
39. $-2^3 = $ 44. $(-\frac{2}{3})^3 = $

$\sqrt{a}$ is a non-negative real number if $a \geq 0$

$\sqrt{a} = b$ means $b^2 = a$, where $b \geq 0$. Thus $\sqrt{49} = 7$, because $7^2 = 49$. Also, $-\sqrt{49} = -7$.

<table>
<thead>
<tr>
<th>Example</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{144} = $</td>
<td>$12$</td>
</tr>
<tr>
<td>$-\sqrt{144} = $</td>
<td>$-12$</td>
</tr>
<tr>
<td>$\sqrt{-144} = $</td>
<td>$i\sqrt{36}$</td>
</tr>
<tr>
<td>$\sqrt{8100} = $</td>
<td>$90$</td>
</tr>
</tbody>
</table>

D. Fraction-decimal conversion:

Fraction to decimal: divide the top by the bottom.

<table>
<thead>
<tr>
<th>Example</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{3} = 3 \div 4 = $</td>
<td>$.75$</td>
</tr>
<tr>
<td>$\frac{20}{3} = 20 \div 3 = $</td>
<td>$6.66666... = \frac{20}{3}$</td>
</tr>
<tr>
<td>$3 \frac{2}{5} = 3 + \frac{2}{5} = $</td>
<td>$3 \div 5 = $</td>
</tr>
</tbody>
</table>

Problems 52-55: Write each as a decimal. If the decimal repeats, show the repeating block of digits:

52. $\frac{5}{8} = $ 54. $4 \frac{1}{3} = $
53. $\frac{3}{7} = $ 55. $\frac{3}{100} = $

Non-repeating decimals to fractions: read the number as a fraction, write it as a fraction, reduce if possible:

<table>
<thead>
<tr>
<th>Example</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$.4 = $ four tenths $= \frac{4}{10} = \frac{2}{5}$</td>
<td></td>
</tr>
<tr>
<td>$3.76 = $ three and seventy six hundredths $= \frac{376}{100} = \frac{188}{50}$</td>
<td></td>
</tr>
</tbody>
</table>

Problems 56-58: Write a fraction:

56. $.01 = $ 57. $.9 = $ 58. $.125 = $

E. Percents:

Meaning of percent: translate ‘percent’ as ‘hundredths’:

<table>
<thead>
<tr>
<th>Example</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8% = $ means $8$ hundredths or $.08$ or $\frac{8}{100} = \frac{2}{25}$</td>
<td></td>
</tr>
</tbody>
</table>

To change a decimal to percent form, multiply by 100: move the point 2 places right and write the % symbol.

<table>
<thead>
<tr>
<th>Example</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$.075 = 7.5%$</td>
<td>$\frac{3}{40} = 7.5%$</td>
</tr>
<tr>
<td>$1 \frac{1}{4} = 1.25 = 125%$</td>
<td></td>
</tr>
</tbody>
</table>

Problems 59-60: Write as a percent:

59. $.3 = $ 60. $.4 = $ 61. $10\% = $ 62. $0.03\% = $

To change a percent to decimal form, move the point 2 places left and drop the % symbol.

<table>
<thead>
<tr>
<th>Example</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8.76% = .0876$</td>
<td>$\frac{876}{10000} = .0876$</td>
</tr>
<tr>
<td>$67% = .67$</td>
<td></td>
</tr>
</tbody>
</table>

Problems 61-62: Write as a decimal:

61. $10\% = $ 62. $0.03\% = $

To solve a percent problem which can be written in this form: $a \%$ of $b$ is $c$

First identify $a, b, c$:

Problems 63-65: If each statement were written (with the same meaning) in the form of $a \%$ of $b$ is $c$, identify $a, b, c$:

63. $3\%$ of $40$ is $1.2$
64. $600$ is $15\%$ of $400$
65. $3$ out of $12$ is $25\%$

Given $a$ and $b$, change $a \%$ to decimal form and multiply (since ‘of’ can be translated ‘multiply’).

Given $c$ and one of the others, divide $c$ by the other (first change percent to decimal, or if the answer is $a$, write it as a percent).
example: What is 9.4% of $5000? (a% of b is c: 
9.4% of $5000 is __?
)
9.4% = .094
.094 × $5000 = $470 (answer)
exmple: 56 problems correct out of 80 is what percent?
(a% of b is c: __?% of 80 is 56)
56 ÷ 80 = .7 = 70% (answer)
exmple: 5610 people vote in an election, which is 60% of the registered voters. How many are registered?
(a % of b is c: 60% of __? is 5610); 60% = .6; 5610 ÷ .6 = 9350 (answer)

66. 4% of 9 is what?
67. What percent of 70 is 56?
68. 15% of what is 60?

F. Estimation and approximation:
Rounding to one significant digit:

example: 3.67 rounds to 4
example: .0449 rounds to .04
example: 850 rounds to either 800 or 900

Problems 69-71: Round to one significant digit.

69. 45.01
70. 1.09
71. 0.0083

To estimate an answer, it is often sufficient to round each given number to one significant digit, then compute.

example: .0298 × .000513
Round and compute: .03 × .0005
=.000015
=.000015 is the estimate

Problems 72-75: Select the best approximation of the answer:

72. 1.2346825 × 367.0003246 =
(4, 40, 400, 4000, 40000)
73. .0042210398 ÷ .0190498238 =
(.02, .2, .5, 5, 20, 50)
74. 101.7283507 + 3.141592653 =
(2, 4, 98, 105, 400)
75. (4.36285903)³ =
(12, 64, 640, 5000, 12000)

Answers:

1. 14
2. 25
3. 34
4. 32
5. 12
6. %, %
7. %, 14
8. 14 (because 20² < 21²)
9. %
10. 1
11. \(\frac{1}{4}\)
12. \(-\frac{5}{3}\)
13. %
14. \(\frac{3}{4}\)
15. %
16. %
17. \(\frac{5}{4}\)
18. 6
19. 15 %
20. \(\frac{7}{8}\)
21. %
22. \(\frac{5}{6}\)
23. 7
24. 6.18
25. -.27

26. 3.791
27. $1.86
28. 32.4
29. .002
30. .2601
31. 2
32. .00013
33. .265
34. 100
35. 9
36. 9
37. -9
38. -9
39. -8
40. 10000
41. 4.41
42. -.001
43. \(\frac{3}{27}\)
44. \(-\frac{5}{27}\)
45. 12
46. -12
47. not a real number
48. 90
49. 1.2
50. .3
51. \(\frac{1}{3}\)
52. .625
53. .428571
54. 43
55. .03
56. \(\frac{1}{100}\)
57. \(\frac{3}{10}\)
58. 4.1 = \(\frac{49}{10}\)
59. 30%
60. 400%
61. .1
62. .0003
63. \(\frac{a}{3}\)
64. 150
65. 25
66. 36
67. 80%
68. 400
69. 50
70. 1
71. .0008
72. 400
73. 2
74. 105
75. 64

TOPIC 2: POLYNOMIALS

A. Grouping to simplify polynomials:
The distributive property says: \(a(b + c) = ab + ac\)

example: \(3(x - y) = 3x - 3y\)
example: $4x + 7x = (4 + 7)x = 11x$
example: $4a + 6x - 2 = 2(2a + 3x - 1)$

Problems 1-3: Rewrite, using the distributive property:
1. $6(x - 3) =$
2. $4x - x =$

Commutative and associative properties are also used in regrouping:

example: $3x + 7 - x = 3x - x + 7 = 2x + 7$
example: $-5 - x = 5 + 5 - x = 10 - x$
example: $3x + 2y - 2x + 3y$

$= 3x - 2x + 2y + 3y = x + 5y$

Problems 4-9: Simplify:
4. $x + x =$
5. $a + b - b + a =$
6. $9x - y + 3y - 8x =$

B. Evaluation by substitution:
example: If $x = 3$, then $7 - 4x = 7 - 4(3) = 7 - 12 = -5$
example: If $a = -7$ and $b = -1$, then $a^2 - b = (-7)^2 - 1 = 49 - 1 = 48$.
example: If $x = -2$, then $3x^2 - x - 5 = 3(-2)^2 - (-2) - 5 = 3(4) - (-2) - 5 = 12 + 2 - 5 = 9$

Problems 10-19: Given $x = -1$, $y = 3$, and $z = -3$. Find the value:
10. $2x =$
11. $-z =$
12. $xz =$
13. $y + z =$
14. $y^2 + z^2 =$

C. Adding, subtracting polynomials:
Combine like terms:
example: $(3x^2 + x + 1) - (x - 1) =$
example: $(x - 1) + (x^2 + 2x - 3) =$
example: $x - 1 + x^2 + 2x - 3 = x^2 + 3x - 4$
example: $(x^2 + x - 1) - (6x^2 - 2x + 1) =$

$= x^2 + x - 1 - 6x^2 + 2x - 1 = -5x^2 + 3x - 2$

Problems 20-25: Simplify:
20. $(x^2 + x) - (x + 1) =$
21. $(x - 3) + (5 - 2x) =$

22. $(2a^2 - a) + (a^2 + a - 1) =$
23. $(y^2 - 2y - 3) - (2y^2 - y + 5) =$
24. $(7 - x) - (x - 7) =$
25. $x^2 - (x^2 + x - 1) =$

D. Monomial times polynomial:
Use the distributive property:
example: $3(x - 4) = 3 \cdot x + 3(-4)$
example: $(2x + 3)a = 2ax + 3a$
example: $-4x(x^2 - 1) = -4x^3 + 4x$

26. $-(x - 7) =$
27. $-2(3 - a) =$
28. $x(x + 5) =$
29. $(3x - 1)^2 =$

E. Multiplying polynomials:
Use the distributive property: $a(b + c) = ab + ac$
example: $(2x + 1)(x - 4)$ is $a(b + c)$ if:
$a = (2x + 1)$, $b = x$, and $c = -4$
So $a(b + c) = ab + ac$

$= (2x + 1)x + (2x + 1)(-4)$
$= 2x^2 + x - 8x + 4 = 2x^2 - 7x - 4$

Short cut to multiply above two binomials (see example above): FOIL (do mentally and write the answer)
F: First times First: $(2x)(x) = 2x^2$
O: multiply ‘Outers’: $(2x)(-4) = -8x$
I: multiply ‘Inners’: $(1)(x) = x$
L: Last times Last: $(1)(-4) = -4$

Add, get $2x^2 - 7x - 4$.

example: $(x + 2)(x + 3) = x^2 + 5x + 6$
example: $(2x - 1)(x + 2) = 2x^2 + 3x - 2$
example: $(x - 5)(x + 5) = x^2 - 25$
example: $-4(x - 3) = -4x + 12$
example: $(3x - 4)^2 = 9x^2 - 24x + 16$
example: $(x + 3)(a - 5) = ax - 5x + 3a - 15$

Problems 33-41: Multiply:
33. $(x + 3)^2 =$
34. $(x - 3)^2 =$
35. $(x + 3)(x - 3) =$
36. $(2x + 3)(2x - 3) =$
37. $(x - 4)(x - 2) =$
38. $-6x(3 - x) =$
39. $(x - \frac{1}{2})^2 =$
40. $(x - 1)(x + 3) =$
41. $(x^2 - 1)(x^2 + 3) =$
F. Special products:
These product patterns (examples of FOIL) should be remembered and recognized:
I. \((a + b)(a - b) = a^2 - b^2\)
II. \((a + b)^2 = a^2 + 2ab + b^2\)
III. \((a - b)^2 = a^2 - 2ab + b^2\)

Problems 42-44: Match each pattern with its example:
a. \((3x - 1)^2 = 9x^2 - 6x + 1\)
b. \((x + 5)^2 = x^2 + 10x + 25\)
c. \((x + 8)(x - 8) = x^2 - 64\)

42. I:
43. II:
44. III:

Problems 45-52: Write the answer using the appropriate product pattern:
45. \((3a + 1)(3a - 1) = \)
46. \((y - 1)^2 = \)
47. \((3a + 2)^2 = \)
48. \((3a + 2)(3a - 2) = \)

49. \((3a - 2)(3a - 2) = \)
50. \((x - y)^2 = \)
51. \((4x + 3y)^2 = \)
52. \((3x + y)(3x - y) = \)

G. Factoring:
Monomial factors: \(ab + ac = a(b + c)\)
example: \(x^2 - x = x(x - 1)\)

Answers:
1. \(6x - 18\)
2. \(3x\)
3. \(-5a + 5\)
4. \(2x\)
5. \(2b\)
6. \(x + 2y\)
7. \(5x - 1\)
8. \(90 - x\)
9. \(-x - y\)
10. \(-2\)
11. \(3\)
12. \(3\)
13. \(0\)
14. \(18\)
15. \(10\)
16. \(2\)
17. \(16\)
18. \(10\)
19. \(3\)
20. \(x^2 - 1\)
21. \(2 - x\)
22. \(3a^2 - 1\)
23. \(-y^2 - 2y - 10\)
24. \(14 - 2x\)
25. \(-x + 1\)
26. \(-x + 7\)
27. \(-6 + 2a\)
28. \(x^2 + 5x\)
29. \(21x - 7\)
30. \(2ax - 3a\)
31. \(-x^2 + 1\)
32. \(24a^2 + 16a - 56\)
33. \(x^2 + 6x + 9\)
34. \(x^2 - 6x + 9\)
35. \(x^2 - 9\)
36. \(4x^2 - 9\)
37. \(x^2 - 6x + 8\)
38. \(-18x + 6x^2\)
39. \(x^2 - x + 1\)
40. \(x^2 + 2x - 3\)
41. \(x^4 + 2x^2 - 3\)
42. \(c\)
43. \(b\)
44. \(a\)
45. \(9a^2 - 1\)
46. \(y^2 - 2y + 1\)
47. \(9a^2 + 12a + 4\)
48. \(9a^2 - 4\)
49. \(9a^2 - 12a + 4\)
50. \(x^2 - 2xy + y^2\)
51. \(16x^2 + 24xy + 9y^2\)
52. \(9x^2 - y^2\)
53. \(a(a + b)\)
54. \(a(a^2 - ab + b^2)\)
55. \(2(2x + 1)(2x - 1)\)
56. \((x - 5)^2\)
57. \(-2x(2y - 5x)\)
58. \((2x - 5)(x + 1)\)
59. \((x - 3)(x + 2)\)
60. \(xy(x - y)\)
61. \((x - 5)(x + 2)\)
62. \(x(2x - 1)\)
63. \(2x(2x + 1)^2\)
64. \((3x + 2)^2\)
65. \(3x^3y(2y - 3x)\)
66. \((1 - 2x)(1 + x)\)
67. \((3x - 1)(x - 3)\)
TOPIC 3: LINEAR EQUATIONS and INEQUALITIES

A. Solving one linear equation in one variable:
Add or subtract the same value on each side of the equation, or multiply or divide each side by the same value, with the goal of placing the variable alone on one side. If there are one or more fractions, it may be desirable to eliminate them by multiplying both sides by the common denominator. If the equation is a proportion, you may wish to cross-multiply.

Problems 1-11: Solve:

1. \(2x = 9\)
2. \(3 = \frac{6x}{5}\)
3. \(3x + 7 = 6\)
4. \(\frac{x}{3} = \frac{5}{4}\)
5. \(5 - x = 9\)
6. \(x = \frac{3x}{5} + 1\)

7. \(4x - 6 = x\)
8. \(x - 4 = \frac{x}{3} + 1\)
9. \(6 - 4x = x\)
10. \(7x - 5 = 2x + 10\)
11. \(4x + 5 = 3 - 2x\)

To solve a linear equation for one variable in terms of the other(s), do the same as above:

**Example:** Solve for \(F: C = \frac{3}{5}(F - 32)\)

Multiply by \(\frac{5}{3}: \frac{5}{3}C = F - 32\)

Add 32: \(\frac{5}{3}C + 32 = F\)

Thus, \(F = \frac{5}{3}C + 32\)

**Example:** Solve for \(b: a + b = 90\)

Subtract \(a: b = 90 - a\)

**Example:** Solve for \(x: ax + b = c\)

Subtract \(b: ax = c - b\)

Divide by \(a: x = \frac{c - b}{a}\)

Problems 12-19: Solve for the indicated variable in terms of the other(s):

12. \(a + b = 180\) \(b = \frac{180 - a}{1}\)
13. \(2a + 2b = 180\) \(b = \frac{180 - 2a}{2}\)
14. \(P = 2b + 2h\) \(b = \frac{P - 2h}{2}\)
15. \(y = 3x - 2\) \(x = \frac{y + 2}{3}\)

B. Solution of a one-variable equation reducible to a linear equation:
Some equations which do not appear to be linear can be solved by using a related linear equation:

**Example:** \(\frac{x + 1}{3x} = -1\)

Multiply by \(3x: x + 1 = -3x\)

Solve: \(4x = -1\)

\(x = -\frac{1}{4}\)

(Be sure to check answer in the original equation.)

**Example:** \(\frac{3x + 3}{x + 1} = 5\)

Think of 5 as \(\frac{5}{1}\) and cross-multiply:

\(5x + 5 = 3x + 3\)

\(2x = -2\)

\(x = -1\)

But \(x = -1\) does not make the original equation true (thus it does not check), so there is no solution.

Problems 20-25: Solve and check:

20. \(\frac{x - 1}{x + 1} = \frac{5}{7}\)
21. \(\frac{3x}{2x + 1} = \frac{5}{2}\)
22. \(\frac{3x - 2}{2x + 1} = 4\)

**Example:** \(|3 - x| = 2\)

Since the absolute value of both 2 and \(-2\) is 2, \(3 - x\) can be either 2 or \(-2\). Write these two equations and solve each:

\(3 - x = 2\) or \(3 - x = -2\)

\(-x = -1\) or \(-x = -5\)

\(x = 1\) or \(x = 5\)

Problems 26-30: Solve:

26. \(|x| = 3\)
27. \(|x| = -1\)
28. \(|x - 1| = 3\)
29. \(|2 - 3x| = 0\)
30. \(|x + 2| = 1\)

C. Solution of linear inequalities:

**Rules for inequalities:**

If \(a > b\), then:

- \(a + c > b + c\)
- \(a - c > b - c\)
- \(ac > bc\) (if \(c > 0\))
- \(ac < bc\) (if \(c < 0\))
- \(\frac{a}{c} > \frac{b}{c}\) (if \(c > 0\))
- \(\frac{a}{c} < \frac{b}{c}\) (if \(c < 0\))

**Example:** One variable graph: solve and graph on a number line: \(1 - 2x \leq 7\) (This is an abbreviation for \(\{x: 1 - 2x \leq 7\}\))

Subtract 1, get \(-2x \leq 6\)

Divide by \(-2\), \(x \geq -3\)

Graph:

Problems 31-38: Solve and graph on a number line:

31. \(x - 3 > 4\)
32. \(4x < 2\)
D. Solving a pair of linear equations in two variables:
The solution consists of an ordered pair, an infinite number of ordered pairs, or no solution.

Problems 39-46: Solve for the common solution(s) by substitution or linear combinations:

<table>
<thead>
<tr>
<th>Answers:</th>
<th>Problems 39-46: Solve for the common solution(s) by substitution or linear combinations:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\frac{3}{2}$</td>
<td>20. 13</td>
</tr>
<tr>
<td>2. $\frac{1}{2}$</td>
<td>21. $-\frac{3}{4}$</td>
</tr>
<tr>
<td>3. $-\frac{3}{5}$</td>
<td>22. $-\frac{5}{3}$</td>
</tr>
<tr>
<td>4. $\frac{3}{4}$</td>
<td>23. 1</td>
</tr>
<tr>
<td>5. $-4$</td>
<td>24. 4</td>
</tr>
<tr>
<td>6. $\frac{3}{5}$</td>
<td>25. no solution</td>
</tr>
<tr>
<td>7. 2</td>
<td>26. $-3, 3$</td>
</tr>
<tr>
<td>8. 10</td>
<td>27. no solution</td>
</tr>
<tr>
<td>9. $\frac{3}{5}$</td>
<td>28. $-2, 4$</td>
</tr>
<tr>
<td>10. 3</td>
<td>29. $\frac{5}{3}$</td>
</tr>
<tr>
<td>11. $-\frac{3}{5}$</td>
<td>30. $-3, -1$</td>
</tr>
<tr>
<td>12. $180 - a$</td>
<td>31. $x &gt; 7$</td>
</tr>
<tr>
<td>13. $90 - a$</td>
<td>32. $x &lt; \frac{1}{2}$</td>
</tr>
<tr>
<td>14. $\frac{(p - 2h)}{3}$</td>
<td>33. $x \leq \frac{5}{3}$</td>
</tr>
<tr>
<td>15. $\frac{(r + 2h)}{3}$</td>
<td>34. $x &gt; 6$</td>
</tr>
<tr>
<td>16. $4 - y$</td>
<td>35. $x &gt; -1$</td>
</tr>
<tr>
<td>17. $\frac{(y - 5)}{2} = \frac{3(y - 1)}{2}$</td>
<td>36. $x &lt; 4$</td>
</tr>
<tr>
<td>18. $\frac{b}{a}$</td>
<td>37. $x &gt; 5$</td>
</tr>
<tr>
<td>19. $\frac{c}{a}$</td>
<td>38. $x \geq -4$</td>
</tr>
<tr>
<td>23.</td>
<td>39. $(9, -1)$</td>
</tr>
<tr>
<td>24. 4</td>
<td>40. $(1, 4)$</td>
</tr>
<tr>
<td>25. no solution</td>
<td>41. $(8, 25)$</td>
</tr>
<tr>
<td>26. $-3, 3$</td>
<td>42. $(-4, -9)$</td>
</tr>
<tr>
<td>27. no solution</td>
<td>43. $(\frac{8}{9}, -\frac{13}{9})$</td>
</tr>
<tr>
<td>28. $-2, 4$</td>
<td>44. $(\frac{1}{4}, 0)$</td>
</tr>
<tr>
<td>29. $\frac{5}{3}$</td>
<td>45. no solution</td>
</tr>
<tr>
<td>30. $-3, -1$</td>
<td>46. any ordered pair of the form $(a, 2a - 3)$ where $a$ is any number. One example: $(4, 5)$.</td>
</tr>
</tbody>
</table>

TOPIC 4: QUADRATIC EQUATIONS

A. $ax^2 + bx + c = 0$:
A quadratic equation can always be written so it looks like $ax^2 + bx + c = 0$ where $a$, $b$, and $c$ are real numbers and $a$ is not zero.

example: $5 - x = 3x^2$
Add: $x = 3x^2 + x$  
Subtract 5: $0 = 3x^2 + x - 5$  
or $3x^2 + x - 5 = 0$  
So $a = 3$, $b = 1$, $c = -5$

example: $x^2 = 3$
Rewrite: $x^2 - 3 = 0$  
(Think of $x^2 + 0x - 3 = 0$)  
So $a = 1$, $b = 0$, $c = -3$

Problems 1-4: Write each of the following in the form $ax^2 + bx + c = 0$ and identify $a, b, c$:

1. $3x + x^2 - 4 = 0$  
2. $5 - x^2 = 0$  
3. $x^2 = 3x - 1$  
4. $x = 3x^2$  
5. $81x^2 = 1$

B. Factoring:
Monomial factors:
$ab + ac = a(b + c)$

example: $x^2 - x = x(x - 1)$

example: $4x^2y + 6xy = 2xy(2x + 3)$

Difference of two squares:
$a^2 - b^2 = (a + b)(a - b)$

example: $9x^2 - 4 = (3x + 2)(3x - 2)$
Trinomial square:
\[ a^2 + 2ab + b^2 = (a + b)^2 \]
\[ a^2 - 2ab + b^2 = (a - b)^2 \]

**example:** \( x^2 - 6x + 9 = (x - 3)^2 \)

Trinomial:

**example:** \( x^2 - x - 2 = (x - 2)(x + 1) \)
**example:** \( 6x^2 - 7x - 3 = (3x + 1)(2x - 3) \)

Problems 6-20: Factor:
6. \( a^2 + ab = \)
7. \( a^3 - a^2 b + ab^2 = \)
8. \( 8x^2 - 2 = \)
9. \( x^2 - 10x + 25 = \)
10. \(-4xy + 10x^2 = \)
11. \( 2x^2 - 3x - 5 = \)
12. \( x^2 - x - 6 = \)
13. \( x^3 y^2 - x^3 y = \)
14. \( x^2 - 3x - 10 = \)
15. \( 2x^2 - x = \)
16. \( 2x^3 + 8x^2 + 8x = \)
17. \( 9x^2 + 12x + 4 = \)
18. \( 6x^3 y^2 - 9x^4 y = \)
19. \( 1 - x - 2x^2 = \)
20. \( 3x^2 - 10x + 3 = \)

C. Solving factored quadratic equations:
The following statement is the central principle:

If \( ab = 0 \), then \( a = 0 \) or \( b = 0 \).

First, identify \( a \) and \( b \) in \( ab = 0 \):

**example:** \( (3-x)(x+2) = 0 \)

Compare this with \( ab = 0 \)
\( a = (3-x) \); \( b = (x+2) \)

Problems 21-24: Identify \( a \) and \( b \) in each of the following:
21. \( 3x(2x - 5) = 0 \)
22. \( (x - 3)x = 0 \)

Then, because \( ab = 0 \) means \( a = 0 \) or \( b = 0 \),
we can use the factors to make two linear equations to solve:

**example:** If \( 2x(3x - 4) = 0 \) then \( 2x = 0 \) or \( 3x - 4 = 0 \)
and so \( x = 0 \) or \( 3x = 4 \); \( x = \frac{4}{3} \).
Thus, there are two solutions: \( 0 \) and \( \frac{4}{3} \)

**example:** If \( (3-x)(x+2) = 0 \) then \( 3-x = 0 \) or \( x+2 = 0 \) and thus \( x = 3 \) or \( x = -2 \).

**example:** If \( (2x+7)^2 = 0 \)
then \( 2x + 7 = 0 \)
\( 2x = -7 \)
\( x = -\frac{7}{2} \) (one solution)

Note: there must be a zero on one side of the equation to solve by the factoring method.

Problems 25-31: Solve:
25. \( (x + 1)(x - 1) = 0 \)
26. \( 4x(x+4) = 0 \)
27. \( 0 = (2x - 5)x \)
28. \( 0 = (2x + 3)(x-1) \)
29. \( (x - 6)(x - 6) = 0 \)
30. \( (2x - 3)^2 = 0 \)
31. \( x(x + 2)(x - 3) = 0 \)

D. Solving quadratic equations by factoring:

Arrange the equation so zero is on one side (in the form \( ax^2 + bx + c = 0 \)), factor, set each factor equal to zero, and solve the resulting linear equations.

**example:** Solve:
\( 6x^2 - 3x = 0 \)
Rewrite:
\( 6x^2 - 3x = 0 \)
Factor:
\( 3x(2x - 1) = 0 \)
So \( 3x = 0 \) or \( (2x - 1) = 0 \)
Thus, \( x = 0 \) or \( x = \frac{1}{2} \)

**example:** \( 0 = x^2 - x - 12 \)
\( 0 = (x - 4)(x + 3) \)
\( x - 4 = 0 \) or \( x + 3 = 0 \)
\( x = 4 \) or \( x = -3 \)

Problems 32-43: Solve by factoring:
32. \( x(x - 3) = 0 \)
33. \( x^2 - 2x = 0 \)
34. \( 2x^2 = x \)
35. \( 3x(x + 4) = 0 \)
36. \( x^2 = 2 - x \)
37. \( x^2 + x = 6 \)

Another problem form: if a problem is stated in this form: ‘One of the solutions of
\( ax^2 + bx + c = 0 \) is \( d \),’ solve the equation as above, then verify the statement.

**example:** Problem: One of the solutions of
\( 10x^2 - 5x = 0 \) is
A. \(-2\)
B. \(-\frac{1}{2}\)
C. \(\frac{1}{2}\)
D. \(2\)
E. \(5\)

Solve \( 10x^2 - 5x = 0 \) by factoring:
\( 5x(2x - 1) = 0 \) so \( 5x = 0 \) or \( (2x - 1) = 0 \)
thus \( x = 0 \) or \( x = \frac{1}{2} \).
Since \( x = \frac{1}{2} \) is one solution, answer C is correct.
44. One of the solutions of \((x - 1)(3x + 2) = 0\) is
   A. \(-\frac{1}{2}\)
   B. \(-\frac{2}{3}\)
   C. 0
   D. \(-\frac{3}{2}\)
   E. \(\frac{3}{2}\)

   **Answers:**
<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>-4</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>81</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

   Note: all signs could be the opposite.

   6. \(a(a + b)\)
   7. \(a^2 - ab + b^2\)
   8. \(2(x + 1)(2x - 1)\)
   9. \((x - 5)^2\)
   10. \(-2x(2y - 5x)\)
   11. \((2x - 5)(x + 1)\)
   12. \((x - 3)(x + 2)\)
   13. \(xy(x - y)\)

   14. \((x - 5)(x + 2)\)
   15. \(x(2x - 1)\)
   16. \(2x(x + 2)^2\)
   17. \((3x + 2)^3\)
   18. \(3x^2y(2y - 3x)\)
   19. \((1 - 2x)(x + 1)\)
   20. \((3x - 1)(x - 3)\)

   21. \(3x\)
   22. \(x - 3\)
   23. \(2x - 1\)
   24. \(x - 1\)
   25. \(-1, 1\)
   26. \(-4, 0\)
   27. \(0, \frac{1}{2}\)
   28. \(-\frac{1}{2}, 1\)

   29. 6
   30. \(\frac{1}{2}\)
   31. \(-2, 0, 3\)
   32. 0, 3
   33. 0, 2
   34. 0, \(\frac{1}{2}\)
   35. \(-4, 0\)
   36. \(-2, 1\)
   37. \(-3, 2\)
   38. \(-2, 3\)
   39. \(-\frac{1}{2}, \frac{1}{2}\)
   40. \(-\frac{1}{2}, \frac{1}{2}\)
   41. 3
   42. \(-1, \frac{1}{2}\)
   43. \(-2, 3\)
   44. B
   45. B

   **TOPIC 5: GRAPHING**

   **A. Graphing a point on the number line:**
   Problems 1-7: Select the letter of the point on the number line with coordinate:

   ![Number Line](image)

   1. 0
   2. \(\frac{1}{2}\)
   3. \(-\frac{1}{2}\)
   4. \(\frac{4}{3}\)
   5. \(-1.5\)
   6. 2.75
   7. \(-\frac{3}{2}\)

   Problems 8-10: Which letter best locates the given number:

   ![Number Line](image)

   8. \(\frac{5}{9}\)
   9. \(\frac{3}{4}\)
   10. \(\frac{2}{3}\)

   Problems 11-13: Solve each equation and graph the solution on the number line:

   **Example:** \(x + 3 = 1\)
   \(x = -2\)

   ![Graph](image)

   11. \(2x - 6 = 0\)
   12. \(x = 3x + 5\)
   13. \(4 - x = 3 + x\)

   **B. Graphing a linear inequality (in one variable) on the number line:**

   Rules for inequalities:
   If \(a > b\), then:
   \(a + c > b + c\)
   \(a - c > b - c\)
   \(ac > bc\) (if \(c > 0\))
   \(ac < bc\) (if \(c < 0\))
   \(\frac{a}{c} > \frac{b}{c}\) (if \(c > 0\))
   \(\frac{a}{c} < \frac{b}{c}\) (if \(c < 0\))

   **Example:** One variable graph: solve and graph on a number line: \(1 - 2x \leq 7\)
   (This is an abbreviation for \(\{x : 1 - 2x \leq 7\}\))
   Subtract 1, get \(-2x \leq 6\)
   Divide by -2, \(x \geq -3\)
   Graph:

   ![Graph](image)
Problems 14-20: Solve and graph on a number line:
14. \(x - 3 > 4\)
15. \(4x < 2\)
16. \(2x + 1 \leq 6\)
17. \(3 < x - 3\)
18. \(4 - 2x < 6\)
19. \(5 - x > x - 3\)
20. \(x > 1 + 4\)

**example:** \(x > -3\) and \(x < 1\)

The two numbers \(-3\) and \(1\) splits the number line into three parts: \(x > -3\), \(-3 < x < 1\), and \(x < 1\). Check each part to see if both \(x > -3\) and \(x < 1\) are true:

<table>
<thead>
<tr>
<th>part</th>
<th>(x) values</th>
<th>(x &gt; -3)?</th>
<th>(x &lt; 1)?</th>
<th>both true?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(x &lt; -3)</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>(-3 &lt; x &lt; 1)</td>
<td>yes</td>
<td>yes</td>
<td>yes (solution)</td>
</tr>
<tr>
<td>3</td>
<td>(x &gt; 1)</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

Thus the solution is \(-3 < x < 1\) and the graph is:

![Graph of x > -3 and x < 1]

**example:** \(x \leq -3\) or \(x < 1\)

(‘or’ means ‘and/or’)

<table>
<thead>
<tr>
<th>part</th>
<th>(x) values</th>
<th>(x \leq -3)?</th>
<th>(x &lt; 1)?</th>
<th>at least one true?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(x \leq -3)</td>
<td>yes</td>
<td>yes</td>
<td>yes (solution)</td>
</tr>
<tr>
<td>2</td>
<td>(-3 \leq x &lt; 1)</td>
<td>yes</td>
<td>yes</td>
<td>yes (solution)</td>
</tr>
<tr>
<td>3</td>
<td>(x &gt; 1)</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

So \(x \leq -3\) or \(-3 \leq x < 1\); these cases are both covered if \(x < 1\). Thus the solution is \(x < 1\) and the graph is:

![Graph of x \leq -3 or x < 1]

Problems 21-23: Solve and graph:
21. \(x < 1\) or \(x > 3\)
22. \(x \geq 0\) and \(x > 2\)
23. \(x > 1\) and \(x \leq 4\)

**C. Graphing a point in the coordinate plane:**

If two number lines intersect at right angles so that:
1) one is horizontal with positive to the right and negative to the left,
2) the other is vertical with positive up and negative down, and
3) the zero points coincide

Then they form a coordinate plane, and
1) the horizontal number line is called the \(x\)-axis,
2) the vertical line is the \(y\)-axis,
3) the common zero point is the origin,
4) there are four quadrants, numbered as shown:

![Coordinate Plane]

To locate a point on the plane, an ordered pair of numbers is used, written in the form \((x, y)\).
The \(x\)-coordinate is always given first.

Problems 24-27: Identify \(x\) and \(y\) in each ordered pair:
24. \((3, 0)\)
25. \((-2, 5)\)
26. \((5, -2)\)
27. \((0, 3)\)

To plot a point, start at the origin and make the moves, first in the \(x\)-direction (horizontal) and the in the \(y\)-direction (vertical) indicated by the ordered pair.

**example:** \((-3, 4)\)

Start at the origin, move left 3 (since \(x = -3\)),
then (from there), up 4 (since \(y = 4\))

Put a dot there to indicate the point \((-3, 4)\)

28. Join the following points in the given order: \((-3, -2), (1, -4), (-3, 0), (2, 3), (-1, 2), (3, 0), (-3, -2), (-1, 2), (1, -4)\)

29. Two of the lines you drew cross each other. What are the coordinates of this crossing point?

30. In what quadrant does the point \((a, b)\) lie, if \(a > 0\) and \(b < 0\)?

Problems 31-34: For each given point, which of its coordinates, \(x\) or \(y\), is larger?

**D. Graphing linear equations on the coordinate plane:**

The graph of a linear equation is a line, and one way to find the line is to join points of the line. Two points determine a line, but three are often plotted on a graph to be sure they are collinear (all in a line).

**Case I:** If the equation looks like \(x = a\), then there is no restriction on \(y\), so \(y\) can be any number. Pick 3 numbers for values of \(y\), and make 3 ordered pairs so each has \(x = a\).
Plot and join.

![Graph of x = a]
example: $x = -2$
Select three $y$'s, say -3, 0, and 1
Ordered pairs: $(-2, -3), (-2, 0), (-2, 1)$
Plot and join:
Note the slope formula gives $\frac{-3-0}{-2-(-2)} = \frac{-3}{0}$,
which is not defined: a vertical line has no slope.

Case II: If the equation looks like $y = mx + b$,
where either $m$ or $b$ (or both) can be zero, select
any three numbers for values of $x$, and find the
Corresponding $y$ values. Graph (plot) these
Ordered pairs and join.

example: $y = -2$
Select three $x$'s, say -1, 0, and 2
Since $y$ must be -2,
the pairs are: $(-1, -2), (0, -2), (2, -2)$
The slope is $\frac{2-(-2)}{-1-0} = \frac{0}{-1} = 0$
And the line is horizontal.

example: $y = 3x - 1$
Select 3 $x$'s, say 0, 1, 2:
If $x = 0$, $y = 3 \cdot 0 - 1 = -1$
If $x = 1$, $y = 3 \cdot 1 - 1 = 2$
If $x = 2$, $y = 3 \cdot 2 - 1 = 5$
Ordered pairs: $(0, -1), (1, 2), (2, 5)$
Note the slope is $\frac{2-(-1)}{1-0} = \frac{3}{1} = 3$,
And the line is neither horizontal nor vertical.

Problems 35-41: Graph each line on the number
plane and find its slope (refer to section E below
if necessary):
35. $y = 3x$
36. $x - y = 3$
37. $x = 1 - y$
38. $y = 1$
39. $x = -2$
40. $y = -2x$
41. $y = \frac{1}{2}x + 1$
42. $\frac{3}{6} = \frac{0-3}{-1-1}$
43. $\frac{5-2}{1-(-1)} = \frac{3}{2}$
44. $\frac{-6-(-1)}{3-10} = -\frac{5}{7}$
45. $\frac{0-(-4)}{1-4} = \frac{5}{3}$
46. $\frac{1-0}{x_2-x_1}$
47. $\frac{2}{3}$

The line joining the points $P_1(x_1, y_1)$ and
$P_2(x_2, y_2)$ has slope $\frac{y_2-y_1}{x_2-x_1}$.

example: $A (3, -1), B (-2, 4)$
slope of $\overline{AB} = \frac{4-(-1)}{-2-3} = \frac{5}{-5} = -1$

Problems 48-52: Find the slope of the line
joining the given points:
48. $(-3, 1)$ and $(-1, -4)$
49. $(0, 2)$ and $(-3, -5)$
50. $(3, -1)$ and $(5, -1)$
51.
52.

Answers:
1. D
2. E
3. C
4. F
5. B
6. G
7. B
8. Q
9. T
10. S
11. 3
12. $-\frac{3}{2}$
13. $\frac{3}{2}$
14. $x > 7$
15. $x < \frac{3}{2}$
16. $x \leq \frac{3}{2}$
17. $x > 6$
18. $x > -1$
19. $x < 4$
20. $x > 5$
21. $x < 1$ or $x > 3$
22. $x > 2$
23. $1 \leq x \leq 4$
24. $x > 0$
25. $x > 3$
26. $x > 5$
27. $x > 1$
28. $x > 2$
29. (0, -1)
30. IV
31. x
32. y
33. y
34. x
35. 3
36. 1
37. -1
38. 0
39. none
40. y
41. x
TOPIC 6: RATIONAL EXPRESSIONS

A. Simplifying fractional expressions:

**example:** $\frac{27}{36} = \frac{3\times9}{3\times12} = \frac{9}{12} = \frac{3}{4}

(note that you must be able to find a common factor—in this case 9—in both the top and bottom in order to reduce a fraction.)

**example:** $\frac{3a}{12ab} = \frac{3a\cdot1}{3a\cdot4b} = \frac{3a}{3a} \cdot \frac{1}{4b} = 1 \cdot \frac{1}{4b} = \frac{1}{4b}$

Problems 1-12: Reduce:

1. $\frac{13}{52} = \frac{1}{4}$
2. $\frac{26}{65} = \frac{2}{5}$
3. $\frac{3+6}{3+9} = \frac{9}{12} = \frac{3}{4}$
4. $\frac{6axy}{15by} = \frac{2xy}{5y}$
5. $\frac{19a^2}{95a^2} = \frac{19}{95} = \frac{1}{5}$
6. $\frac{14x-7y}{7y} = \frac{2x-1}{y}$

**example:** $\frac{3}{x} \cdot \frac{y}{15} = \frac{3\cdot10\cdot10}{x\cdot15\cdoty^2} = \frac{300}{15xy^2} = \frac{20}{5xy^2}$

Problems 13-14: Simplify:

13. $\frac{4x^3}{6} \cdot \frac{xy^2}{3y^2} = \frac{4x^3y^2}{6y^2} = \frac{2x^3}{3}$
14. $\frac{x^7-3x^4}{2x^3-6} = \frac{1}{2}$

B. Evaluation of fractions:

**example:** If $a = -1$ and $b = 2$,

find the value of $\frac{a+3}{2b-1}$

Substitute: $\frac{-1+3}{2(2)-1} = \frac{2}{3}$

Problems 15-22: Find the value, given $a = -1$, $b = 2$, $c = 0$, $x = -3$, $y = 1$, $z = 2$:

15. $\frac{5}{b} = \frac{5}{2}$
16. $\frac{x}{a} = \frac{-1}{3}$
17. $\frac{a}{3} = \frac{-1}{2}$
18. $\frac{a-y}{b} = \frac{-3}{2}$

C. Equivalent fractions:

**example:** $\frac{3}{4}$ is equivalent to how many eighths?

$\frac{3}{4} = \frac{8}{8} = \frac{1}{3} \cdot \frac{2}{4} = \frac{2\cdot3}{2\cdot4} = \frac{6}{8}$

**example:** $\frac{6}{5a} = \frac{3ab}{5ab}$

$\frac{6}{5a} = \frac{b}{5} \cdot \frac{6}{5a}$

**example:** $\frac{3x+2}{x+1} = \frac{4(x+1)}{4(x+1)}$

**example:** $\frac{x-1}{x+1} = \frac{(x-2)(x-1)}{(x-2)(x+1)} = \frac{x^2-3x+2}{(x+1)(x-2)}$

Problems 23-27: Complete:

23. $\frac{4}{9} = \frac{72}{27}$
24. $\frac{3x}{y} = \frac{72}{27}$
25. $\frac{x^2-1}{x+2} = \frac{x^2-1}{x+2}$

How to get the lowest common denominator (LCD) by finding the least common multiple (LCM) of all denominators:

**example:** $\frac{2}{6}$ and $\frac{5}{15}$. First find LCM of 6 and 15:

$6 = 2 \cdot 3$
$15 = 3 \cdot 5$

LCM = $2 \cdot 3 \cdot 5 = 30$, so

$\frac{2}{6} = \frac{25}{30}$ and $\frac{5}{15} = \frac{2}{15}$

**example:** $\frac{3}{x} + \frac{1}{6a}$:

$4 = 2 \cdot 2$
$6a = 2 \cdot 3 \cdot a$

LCM = $2 \cdot 3 \cdot a = 12a$, so

$\frac{4}{6} = \frac{9a}{12a}$ and $\frac{1}{6a} = \frac{2}{12a}$

**example:** $\frac{5}{x+2}$ and $-\frac{1}{x-2}$

$\text{LCM} = (x+2)(x-2)$, so

$\frac{5(x-2)}{(x+2)(x-2)}$, and $-\frac{1}{x-2} = \frac{-1(x+2)}{(x+2)(x-2)}$

Problems 28-33: Find equivalent fractions with the lowest common denominator:

28. $\frac{2}{3}$ and $\frac{2}{9}$
29. $\frac{3}{5}$ and 5
30. $\frac{2}{3}$ and $\frac{4}{x+1}$
31. $\frac{3}{x-2}$ and $\frac{4}{2-x}$
32. $\frac{4}{x-3}$ and $\frac{5}{x+5}$
33. $\frac{1}{x}$ and $\frac{3x}{x+1}$
D. Adding and subtracting fractions:
If denominators are the same, combine the numbers:

\[
\text{example: } \frac{3x}{y} - \frac{x}{y} = \frac{3x-x}{y} = \frac{2x}{y}
\]

Problems 34-38: Find the sum or difference as indicated (reduce if possible):

34. \(\frac{4}{7} + \frac{2}{7} = \)
35. \(\frac{3}{x-3} - \frac{x}{x-3} = \)
36. \(\frac{b-a}{b+a} - \frac{a-b}{b+a} = \)

If denominators are different, find equivalent fractions with common denominators, then proceed as before (combine numerators):

\[
\text{example: } \frac{3}{x-1} + \frac{1}{x+2} = \frac{3(x+2)}{(x-1)(x+2)} + \frac{x-1}{(x-1)(x+2)} = \frac{3x+6+x-1}{(x-1)(x+2)} = \frac{4x+5}{(x-1)(x+2)}
\]

Problems 39-51: Find the sum or difference:

39. \(\frac{\frac{3}{a} - \frac{1}{a}}{=\)\)
40. \(\frac{\frac{3}{x} - \frac{\frac{2}{x}}{y}}{=\)\)
41. \(\frac{\frac{4}{x} - \frac{\frac{2}{x}}{y}}{=\)\)
42. \(\frac{\frac{2}{x} + \frac{2}{x}}{=\)\)
43. \(\frac{\frac{\frac{a}{b} - \frac{2}{b}}{=\)\)
44. \(\frac{\frac{a}{x} + \frac{2}{b}}{=\)\)
45. \(\frac{\frac{1}{a} + \frac{1}{b}}{=\)\)

E. Multiplying fractions:

Multiply the tops, multiply the bottoms, reduce if possible:

\[
\text{example: } \frac{\frac{2}{3} \cdot \frac{2}{5}}{=\)\}
\]

Answers:

1. \(\frac{1}{4}\)
2. \(\frac{3}{4}\)
3. \(\frac{3}{4}\)
4. \(\frac{\frac{2x}{5b}}{=\)\)
5. \(\frac{\frac{3}{5}}{=\)\)
6. \(\frac{\frac{2x}{y}}{=\)\)
7. \(\frac{\frac{3a+b}{4a+5c}}{=\)\)
8. \(-1\)
9. \(\frac{\frac{2(\frac{x}{x}+1)}{=\)\}
10. \(x\)
11. \(\frac{\frac{4(x-1)}{3(x+1)}}{=\)\)
12. \(\frac{\frac{2x+1}{x+1}}{=\)\)
13. \(x^2\)
14. \(x^2\)
15. \(3\)
16. \(3\)
17. \(-1\)
18. \(-1\)
19. \(-\frac{1}{2}\)
20. undefined
21. -1
22. 0
23. 32
24. 3xy
25. \(x^2+2x-3\) or \((x-1)(x+3)\)
26. \(2+2b-a-ab\) or \((1+b)(2-a)\)
27. 2
28. \(\frac{6}{5}, \frac{2}{5}\)
29. \(\frac{2}{3}, \frac{2}{3}\)
30. \(\frac{\frac{x(x+1)}{3(x+1)} + \frac{4}{5(x+2)}}{=\)\)
31. \(\frac{\frac{1}{x^2} + \frac{1}{x^2}}{=\)\
### TOPIC 7: EXPONENTS and SQUARE ROOT

#### A. Positive integer exponents:
\(a^b\) means use \(a\) as a factor \(b\) times. \((b\) is the exponent or power of \(a\).

**example:** \(2^5\) means \(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2\), and has value 32.

**example:** \(c \cdot c \cdot c = c^3\)

Problems 1-14: Find the value:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(\frac{-4(x+3)}{(x-3)(x+3)})</td>
<td>45.</td>
</tr>
<tr>
<td>2.</td>
<td>(\frac{x+1}{x(x+1)})</td>
<td>46.</td>
</tr>
<tr>
<td>3.</td>
<td>(\frac{-5(x-3)}{(x-3)(x+3)})</td>
<td>47.</td>
</tr>
<tr>
<td>4.</td>
<td>(\frac{3x^2}{x(x+1)})</td>
<td>48.</td>
</tr>
<tr>
<td>5.</td>
<td>(\frac{x+1}{x+1})</td>
<td>49.</td>
</tr>
<tr>
<td>6.</td>
<td>(\frac{x}{x^2+2})</td>
<td>50.</td>
</tr>
<tr>
<td>7.</td>
<td>(\frac{\sqrt{x}}{x})</td>
<td>51.</td>
</tr>
<tr>
<td>8.</td>
<td>(\frac{\sqrt{b}}{b})</td>
<td>52.</td>
</tr>
<tr>
<td>9.</td>
<td>(\frac{\sqrt{a}}{b})</td>
<td>53.</td>
</tr>
<tr>
<td>10.</td>
<td>(\frac{\sqrt{c}}{c})</td>
<td>54.</td>
</tr>
<tr>
<td>11.</td>
<td>(\frac{\sqrt{d}}{d})</td>
<td>55.</td>
</tr>
<tr>
<td>12.</td>
<td>(\frac{\sqrt{e}}{e})</td>
<td>56.</td>
</tr>
<tr>
<td>13.</td>
<td>(\frac{\sqrt{f}}{f})</td>
<td>57.</td>
</tr>
</tbody>
</table>

**example:** Simplify:

\(a \cdot a \cdot a \cdot a = a^5\)

Problems 15-18: Simplify:

15. \(3^2 \cdot x^4\)  
16. \(2^4 \cdot b \cdot b \cdot b \cdot b\)

**B. Integer exponents:**

1. \(a^b \cdot a^c = a^{b+c}\)
2. \(\frac{a^b}{a^c} = a^{b-c}\)
3. \((a^b)^c = a^{bc}\)
4. \((ab)^c = a^c \cdot b^c\)
5. \((b^c)^a = a^c \cdot b^c\)

**C. Scientific notation:**

**example:** 32800 = 3.2800 \times 10^4 if the zeros in the ten’s and one’s places are significant. If the one’s zero is not, write 3.280 \times 10^4; if neither is significant: 3.28 \times 10^4.

**example:** .004031 = 4.031 \times 10^{-3}
example: \(2 \times 10^2 = 200\)
example: \(9.9 \times 10^{-1} = 0.99\)

Note that scientific form always looks like \(a \times 10^n\) where \(1 \leq a < 10\), and \(n\) is an integer power of 10.

Problems 42-45: Write in scientific notation:
42. \(93,000,000 = \)
44. \(5.07 = \)
43. \(.000042 = \)
45. \(-32 = \)

Problems 46-48: Write in standard notation:
46. \(1.4030 \times 10^3 = \)
48. \(4 \times 10^{-6} = \)
47. \(-9.11 \times 10^{-2} = \)

To compute with numbers written in scientific form, separate the parts, compute, then recombine.

example: \((3.14 \times 10^5) \times 2 =\)
\((3.14) \times (2) \times 10^5 = 6.28 \times 10^5\)

example: \(\frac{4.28 \times 10^6}{2.14 \times 10^3} =\)
\(2.00 \times 10^3\)

example: \(\frac{2.01 \times 10^{-3}}{8.04 \times 10^{-6}} =\)
\(0.250 \times 10^3 = 2.50 \times 10^2\)

Problems 49-56: Write in scientific notation:
49. \(10^{40} \times 10^{-2} = \)
53. \(\frac{1.8 \times 10^{-4}}{3.6 \times 10^{-6}} =\)

50. \(\frac{10^{40}}{10^{18}} =\)
54. \((4 \times 10^{-3})^2 =\)

51. \(\frac{1.86 \times 10^4}{3 \times 10^{-1}} =\)
55. \((2.5 \times 10^2)^{-1} =\)

52. \(\frac{3.6 \times 10^5}{1.8 \times 10^{-5}} =\)
56. \(-\frac{2.92 \times 10^7}{6.1 \times 10^6} =\)

D. Simplification of square roots:
\(\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}\) if \(a\) and \(b\) are both non-negative \((a \geq 0 \text{ and } b \geq 0)\).

example: \(\sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}\)
example: \(\sqrt{75} = \sqrt{5} \cdot \sqrt{25} = \sqrt{3} \cdot 5 = 5\sqrt{3}\)
example: If \(x \geq 0\), \(\sqrt{x^6} = x^3\)
If \(x < 0\), \(\sqrt{x^6} = |x|^3\)

Note: \(\sqrt{a} = b\) means (by definition) that
1) \(b^2 = a\), and
2) \(b \geq 0\)

Problems 57-69: Simplify (assume all square roots are real numbers):
57. \(\sqrt{81} = \)
59. \(2\sqrt{9} = \)
58. \(-\sqrt{81} = \)
60. \(4\sqrt{9} = \)

61. \(\sqrt{40} = \)
63. \(\sqrt{52} = \)
64. \(\sqrt[9]{16} = \)
65. \(\sqrt{0.09} = \)

E. Adding and subtracting square roots:
example: \(\sqrt{5} + 2\sqrt{5} = 3\sqrt{5}\)
example: \(\sqrt{32} - \sqrt{2} = 4\sqrt{2} - \sqrt{2} = 3\sqrt{2}\)

Problems 70-73: Simplify:
70. \(\sqrt{5} + \sqrt{5} = \)
72. \(\sqrt{2} + \sqrt{2} = \)
71. \(2\sqrt{3} + \sqrt{27} - \sqrt{75} = \)
73. \(5\sqrt{3} - \sqrt{75} = \)

F. Multiplying square roots:
\(\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}\) if \(a \geq 0\) and \(b \geq 0\).

example: \(\sqrt{6} \cdot \sqrt{24} = \sqrt{6} \cdot 24 = \sqrt{144} = 12\)
example: \(\sqrt{2} \cdot \sqrt{6} = \sqrt{12} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}\)
example: \((5\sqrt{2}) \cdot (3\sqrt{2}) = 15\sqrt{4} = 15 \cdot 2 = 30\)

Problems 74-79: Simplify:
74. \(\sqrt{3} \cdot \sqrt{3} = \)
77. \((\sqrt{9})^2 = \)
75. \(\sqrt{3} \cdot \sqrt{4} = \)
78. \((\sqrt{5})^2 = \)
76. \((2\sqrt{3})(3\sqrt{2}) = \)
79. \((\sqrt{3})^4 = \)

Problems 80-81: Find the value of \(x\):
80. \(\sqrt{4} \cdot \sqrt{9} = \sqrt{x} \)
81. \(3\sqrt{2} \cdot \sqrt{5} = 3\sqrt{x} \)

G. Dividing square roots:
\(\sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}\), if \(a \geq 0\) and \(b > 0\).

example: \(\sqrt{5} \div \sqrt{64} = \frac{\sqrt{5}}{\sqrt{64}} = \frac{\sqrt{5}}{8}\) (or \(\frac{1}{8}\sqrt{5}\))

Problems 82-86: Simplify:
82. \(\frac{\sqrt{3}}{\sqrt{4}} = \)
85. \(\sqrt{36} \div 4 = \)
83. \(\frac{\sqrt{3}}{\sqrt{25}} = \)
86. \(\frac{8}{\sqrt{16}} = \)
84. \(\frac{\sqrt{49}}{2} = \)

If a fraction has a square root on the bottom, it is sometimes desirable to find an equivalent fraction with no root on the bottom. This is called rationalizing the denominator.

example: \(\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{6}}{2}\)
example: \(\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{3}\)
### Problems 87-94: Simplify:

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<tr>
<th>Problem</th>
<th>Expression</th>
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</thead>
<tbody>
<tr>
<td>87.</td>
<td>$\sqrt{\frac{9}{4}}$</td>
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<tr>
<td>88.</td>
<td>$\sqrt[3]{\frac{18}{9}}$</td>
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<td>89.</td>
<td>$\frac{\sqrt{4}}{9}$</td>
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<td>90.</td>
<td>$\frac{5}{\sqrt{2}}$</td>
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<td>$\frac{5}{\sqrt{3}}$</td>
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<td>$\frac{\sqrt{a}}{\sqrt{b}}$</td>
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<td>94.</td>
<td>$\sqrt{2} + \frac{1}{\sqrt{2}}$</td>
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</table>

### Answers:

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<th>Problem</th>
<th>Expression</th>
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<tbody>
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<td>8</td>
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<td>$\frac{16}{81}$</td>
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<tr>
<td>41.</td>
<td>-8$x^3y^6$</td>
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<tr>
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<td>$9.3 \times 10^7$</td>
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<td>$4.2 \times 10^{-5}$</td>
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<tr>
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<td>50.</td>
<td>$1 \times 10^{-30}$</td>
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<td>$6.2 \times 10^4$</td>
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<td>$2.0 \times 10^3$</td>
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<tr>
<td>53.</td>
<td>$5.0 \times 10^{-4}$</td>
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<td>$1.6 \times 10^{-5}$</td>
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<td>55.</td>
<td>$4.0 \times 10^{-3}$</td>
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<td>56.</td>
<td>$1.46 \times 10^{13}$</td>
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<td>60.</td>
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</tr>
<tr>
<td>61.</td>
<td>$2\sqrt{10}$</td>
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<tr>
<td>62.</td>
<td>$6\sqrt{3}$</td>
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<tr>
<td>63.</td>
<td>$2\sqrt{13}$</td>
</tr>
<tr>
<td>64.</td>
<td>$\frac{3}{4}$</td>
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<tr>
<td>65.</td>
<td>.3</td>
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</tbody>
</table>
TOPIC 8: GEOMETRY

A. Formulas for perimeter $P$ and area $A$ of rectangles, squares, parallelograms, and triangles:

**Rectangle** with base $b$ and altitude (height) $h$:

$P = 2b + 2h$

$A = bh$

If a wire is bent in this shape, the perimeter $P$ is the length of the wire, and the area $A$ is the number of square units enclosed by the wire.

**Example:** A rectangle with $b = 7$ and $h = 8$:

$P = 2b + 2h = 2 \cdot 7 + 2 \cdot 8 = 14 + 16 = 30$ units

$A = bh = 7 \cdot 8 = 56$ square units

A **square** is a rectangle with all sides equal, so the rectangle formulas apply (and simplify). If the side length is $s$:

$P = 4s$

$A = s^2$

**Example:** A square with side $s = 11 \text{ cm}$ has

$P = 4s = 4 \times 11 = 44 \text{ cm}$

$A = s^2 = 11^2 = 121 \text{ cm}^2$ (sq. cm)

A **parallelogram** with base $b$ and height $h$ and other side $a$:

$A = bh$

$P = 2a + 2b$

**Example:** A parallelogram has sides 4 and 6; 5 is the length of the altitude perpendicular to the side 4.

$P = 2a + 2b = 2 \cdot 6 + 2 \cdot 4 = 12 + 8 = 20 \text{ units}$

$A = bh = 4 \cdot 5 = 20 \text{ square units}$

In a **triangle** with side lengths $a$, $b$, and $c$, and altitude height $h$ to side $b$:

$P = a + b + c$

$A = \frac{1}{2}bh = \frac{bh}{2}$
example:

\[ P = a + b + c \]
\[ = 6 + 8 + 10 \]
\[ = 24 \text{ units} \]

\[ A = \frac{1}{2}bh = \frac{1}{2}(10)(4.8) = 24 \text{ square units} \]

Problems 1-8: Find \( P \) and \( A \):
1. Rectangle with sides 5 and 10.
2. Rectangle with sides 1.5 and 4.
3. Square with sides 3 miles.
4. Square with sides \( \frac{3}{4} \) yards.
5. Parallelogram with sides 36 and 24, and height 10 (on side 36).
6. Parallelogram, all sides 12, altitude 6.
7. Triangle with sides 5, 12, and 13.
   Side 5 is the altitude on side 12.
8. Triangle shown:

B. Formulas for circumference \( C \) and area \( A \) of a circle:

A circle with radius \( r \)
(and diameter \( d = 2r \))
has a distance around
(circumference) \( C = \pi d = 2\pi r \)
(If a piece of wire is bent into a circular shape, the circumference is the length of the wire.)

\[ r \]
\[ d = 2r = 140 \text{ and exact circumference} \]
\[ C = 2\pi r = 2 \cdot \pi \cdot 70 = 140\pi \text{ units} \]
If \( \pi \) is approximated by \( \frac{22}{7} \),
\[ C = 140\pi = 140 \left( \frac{22}{7} \right) = 440 \text{ units (approx.)} \]
If \( \pi \) is approximated by 3.1,
\[ C = 140(3.1) = 434 \text{ units} \]
The area of a circle is \( A = \pi r^2 \)
example: If \( r = 8 \), exact area is
\[ A = \pi r^2 = \pi \cdot 8^2 = 64\pi \text{ square units} \]

Problems 9-11: Find the exact \( C \) and \( A \) for a circle with:
9. radius \( r = 5 \) units
10. \( r = 10 \) feet
11. diameter \( d = 4 \) km

Problems 12-14: A circle has area \( 49\pi \):
12. What is its radius length?
13. What is the diameter?
14. Find its circumference.

Problems 15-16: A parallelogram has area 48 and two sides each of length 12:
15. How long is the altitude to those sides?
16. How long are each of the other two sides?
17. How many times the \( P \) and \( A \) of a 3cm square are the \( P \) and \( A \) of a square with sides all 6 cm?
18. A rectangle has area 24 and one side 6. Find the perimeter.

Problems 19-20: A square has perimeter 30:
19. How long is each side?
20. What is its area?
21. A triangle has base and height each 7. What is its area?

C. Pythagorean theorem:
In any triangle with a \( 90^\circ \) (right) angle, the sum of the squares of the legs equals the square of the hypotenuse.

(The legs are the two shorter sides; the hypotenuse is the longest side.)

If the legs have lengths \( a \) and \( b \), and \( c \) is the hypotenuse length, then \( a^2 + b^2 = c^2 \).
In words: “In a right triangle, leg squared plus leg squared equals hypotenuse squared.”

\[ a^2 + b^2 = c^2 \]
example: A right triangle has hypotenuse 5 and one leg 3. Find the other leg. Since \( leg^2 + leg^2 = hyp^2 \),
\[ 3^2 + x^2 = 5^2 \]
\[ 9 + x^2 = 25 \]
\[ x^2 = 25 - 9 = 16 \]
\[ x = \sqrt{16} = 4 \]

Problems 22-24: Find the length of the third side of the right triangle:
22. one leg: 15, hypotenuse: 17
23. hypotenuse: 10, one leg: 8
24. legs: 5 and 12

Problems 25-26: Find \( x \):

25. \[ x \]
26. \[ x \]
27. \[ x \]
28. In right \( \triangle RST \) with right angle \( R \), \( SR = 11 \) and \( TS = 61 \). Find \( RT \). (Draw and label a triangle to solve.)
29. Would a triangle with sides 7, 11, and 13 be a right triangle? Why or why not?

Similar triangles are triangles which are the same shape. If two angles of one triangle are equal respectively to two angles of another triangle, then the triangles are similar.

**Example:** \( \triangle ABC \) and \( \triangle FED \) are similar:

The pairs of sides which correspond are \( AB \) and \( FE \), \( BC \) and \( BC \), \( AC \) and \( FD \).

Problems 30-32: Use this figure:

30. Find and name two similar triangles.
31. Draw the triangles separately and label them.
32. List the three pairs of corresponding sides.

If two triangles are similar, any two corresponding sides have the same ratio (fraction value):

**Example:** the ratio \( \frac{a}{x} \) to \( \frac{b}{y} \), or \( \frac{a}{x} = \frac{b}{y} \), is the same as \( \frac{c}{z} \) and \( \frac{d}{w} \).

Thus \( \frac{x}{a} = \frac{y}{b} \), \( \frac{a}{x} = \frac{b}{y} \), and \( \frac{c}{z} = \frac{d}{w} \). Each of these equations is called a proportion.

33. Draw the similar triangles separately, label them, and write proportions for the corresponding sides.

Problems 34-37: Solve for \( x \):

**Example:** Find \( x \) by writing and solving a proportion:

\[ \frac{2}{5} = \frac{3}{x}, \] \ so cross multiply and get \( 2x = 15 \) or \( x = 7 \frac{1}{2} \)

34. \( AC = 10; \ EC = 7 \)
35. \( BC = 4; \ DC = x \)
36. \( 10 \)
37. \( 4 \)

**D. Graphing on the number line:**

Problems 38-45: Name the point with given coordinate:

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( D )</th>
<th>( E )</th>
<th>( F )</th>
<th>( G )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

38. \( 0 \)
39. \( \frac{1}{2} \)
40. \( -\frac{1}{2} \)
41. \( \frac{4}{3} \)
42. \( -1.5 \)
43. \( 2.75 \)
44. \( -\frac{3}{2} \)
45. \( 1.3 \)

Problems 46-51: On the number line above, what is the distance between the listed points? (Remember that distance is always positive.)

46. \( D \) and \( G \)
47. \( A \) and \( D \)
48. \( A \) and \( F \)
49. \( B \) and \( C \)
50. \( B \) and \( E \)
51. \( F \) and \( G \)

Problems 52-55: On the number line, find the distance from:

52. \( -7 \) to \( -4 \)
53. \( -7 \) to \( 4 \)
54. \( -4 \) to \( 7 \)
55. \( 4 \) to \( 7 \)

Problems 56-59: Draw a sketch to help find the coordinate of the point…:

56. Halfway between points with coordinates 4 and 14.
57. Midway between \( -5 \) and \( -1 \).
58. Which is the midpoint of the segment joining \( -8 \) and 4.
59. On the number line the same distance from \( -6 \) as it is from \( 10 \).

**E. Coordinate plane graphing:**

To locate a point on the plane, an ordered pair of numbers is used, written in the form \((x, y)\).

Problems 60-63: Identify coordinates \( x \) and \( y \) in each ordered pair:

60. \((3,0)\)
61. \((-2,5)\)
62. \((5,-2)\)
63. \((0,3)\)

To plot a point, start at the origin and make the moves, first in the \( x \)-direction (horizontal) and
then the y-direction (vertical) indicated by the ordered pair.

**example:** \((-3, 4)\)

Start at the origin, move left 3 (since \(x = -3\)),

then (from there), up 4 (since \(y = 4\)),

put a dot there to indicate the point \((-3, 4)\)

64. On graph paper, join these points in order:
\((-3, -2), (1, -4), (3, 0), (2, 3), (-1, 2), (3, 0), (-3, -2), (-1, 2), (1, -4)\).

65. Two of the lines drawn in problem 64 cross each other. What are the coordinates of the crossing point?

66. In what quadrant is the point \((a, b)\) if \(a > 0\) and \(b < 0\)?

Problems 67-69: \(ABCD\) is a square, with \(C(5, -2)\) and \(D(-1, -2)\). Find:

67. the length of each side.
68. the coordinates of \(A\).
69. the coordinates of the midpoint of \(DC\).

Problems 70-72: Given \(A(0, 5), B(12, 0)\):

70. Sketch a graph. Draw \(AB\). Find its length.
71. Find the midpoint of \(AB\) and label it \(C\).
    Find the coordinates of \(C\).
72. What is the area of the triangle formed by \(A, B,\) and the origin?

**Answers:**

| 1. \(30\) units, \(50\) units\(^2\)  
  (units\(^2\) means square units) |
<table>
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<tbody>
<tr>
<td>2. (11) units, (6) units(^2)</td>
</tr>
<tr>
<td>3. (12) miles, (9) miles(^2)</td>
</tr>
<tr>
<td>4. (3) yards, (\frac{9}{16}) yards(^2)</td>
</tr>
<tr>
<td>5. (120) un., (360) un.(^2)</td>
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<tr>
<td>6. (48) un., (72) un.(^2)</td>
</tr>
<tr>
<td>7. (30) un., (30) un.(^2)</td>
</tr>
<tr>
<td>8. (12) un., (6) un.(^2)</td>
</tr>
<tr>
<td>9. (10\pi) un., (25\pi) un.(^2)</td>
</tr>
<tr>
<td>10. (20\pi) ft., (100\pi) ft.(^2)</td>
</tr>
<tr>
<td>11. (4\pi) km, (4\pi) km(^2)</td>
</tr>
<tr>
<td>12. (7)</td>
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<tr>
<td>13. (14)</td>
</tr>
<tr>
<td>14. (14\pi)</td>
</tr>
<tr>
<td>15. (4)</td>
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<tr>
<td>16. Cannot tell</td>
</tr>
<tr>
<td>17. (P) is (2) times, (A) is (4) times</td>
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<td>18. (20)</td>
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<td>19. (7\frac{1}{2})</td>
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<tr>
<td>20. (\frac{225}{4})</td>
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<tr>
<td>21. (24\frac{1}{2})</td>
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<td>22. (8)</td>
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<tr>
<td>23. (6)</td>
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<td>24. (13)</td>
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<td>25. (9)</td>
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<tr>
<td>26. (41)</td>
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<td>27. (10)</td>
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<tr>
<td>28. (60)</td>
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<tr>
<td>29. No, because (7^2 + 11^2 \neq 13^2)</td>
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<tr>
<td>30. (\triangle ABE \sim \triangle ACD)</td>
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<tr>
<td>31.</td>
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<tr>
<td>32. (\overline{AB}, \overline{AC} ; \overline{AE}, \overline{AD} ; \overline{BE}, \overline{CD} )</td>
</tr>
<tr>
<td>33. (\frac{3}{9} = \frac{5}{15} = \frac{4}{12})</td>
</tr>
<tr>
<td>34. (\frac{14}{5}) or (\frac{24}{5})</td>
</tr>
<tr>
<td>35. (\frac{2}{3}) or (\frac{14}{8})</td>
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<tr>
<td>36. (\frac{45}{2})</td>
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<td>37. (\frac{49}{7})</td>
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<td>38. (D)</td>
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<td>39. (E)</td>
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<td>40. (C)</td>
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<td>45. (F)</td>
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<tr>
<td>46. 2.75</td>
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<td>47. 2</td>
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<tr>
<td>48. 3(\frac{1}{3})</td>
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<td>49. 1</td>
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<td>50. 2</td>
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<tr>
<td>51. (\frac{12}{12})</td>
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<td>52. 3</td>
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<td>53. 11</td>
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<tr>
<td>54. 11</td>
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<td>55. 3</td>
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<tr>
<td>56. 9</td>
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<tr>
<td>57. 3</td>
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<td>58. 2</td>
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<tr>
<td>59. 2</td>
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<tr>
<td>60. (x = 3, y = 0)</td>
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<tr>
<td>61. (x = -2, y = 5)</td>
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<tr>
<td>62. (x = 5, y = -2)</td>
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<tr>
<td>63. (x = 0, y = 3)</td>
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<td>64.</td>
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<tr>
<td>65. (0, -1)</td>
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<td>66. IV</td>
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<tr>
<td>67. 6</td>
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<tr>
<td>68. ((-1, 4))</td>
</tr>
<tr>
<td>69. ((2, -2))</td>
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<tr>
<td>70. 13</td>
</tr>
<tr>
<td>71. ((6, \frac{3}{2}))</td>
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<tr>
<td>72. 30</td>
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