Frieze Patterns

**GOAL 1** CLASSIFYING FRIEZE PATTERNS

A **frieze pattern** or **border pattern** is a pattern that extends to the left and right in such a way that the pattern can be mapped onto itself by a horizontal translation. In addition to being mapped onto itself by a horizontal translation, some frieze patterns can be mapped onto themselves by other transformations.

1. Translation \( T \)
2. 180° rotation \( R \)
3. Reflection in a horizontal line \( H \)
4. Reflection in a vertical line \( V \)
5. Horizontal glide reflection \( G \)

**EXAMPLE 1** Describing Frieze Patterns

Describe the transformations that will map each frieze pattern onto itself.

a. [Image of a frieze pattern]

b. [Image of a frieze pattern]

c. [Image of a frieze pattern]

d. [Image of a frieze pattern]

**SOLUTION**

a. This frieze pattern can be mapped onto itself by a horizontal translation \( (T) \).

b. This frieze pattern can be mapped onto itself by a horizontal translation \( (T) \) or by a 180° rotation \( (R) \).

c. This frieze pattern can be mapped onto itself by a horizontal translation \( (T) \) or by a horizontal glide reflection \( (G) \).

d. This frieze pattern can be mapped onto itself by a horizontal translation \( (T) \) or by a reflection in a vertical line \( (V) \).
To classify a frieze pattern into one of the seven categories, you first decide whether the pattern has 180° rotation. If it does, then there are three possible classifications: TR, TRVG, and TRHVG.

If the frieze pattern does not have 180° rotation, then there are four possible classifications: T, TV, TG, and THG. Decide whether the pattern has a line of reflection. By a process of elimination, you will reach the correct classification.

### STUDENT HELP

**Study Tip**

To help classify a frieze pattern, you can use a process of elimination. This process is described at the right and in the tree diagram in Ex. 53.

### EXAMPLE 2  **Classifying a Frieze Pattern**

**SNAKES**  Categorize the snakeskin pattern of the mountain adder.

**SOLUTION**

This pattern is a TRHVG. The pattern can be mapped onto itself by a translation, a 180° rotation, a reflection in a horizontal line, a reflection in a vertical line, and a horizontal glide reflection.
GOAL 2 USING FRIEZE PATTERNS IN REAL LIFE

EXAMPLE 3 Identifying Frieze Patterns

ARCHITECTURE The frieze patterns of ancient Doric buildings are located between the cornice and the architrave, as shown at the right. The frieze patterns consist of alternating sections. Some sections contain a person or a symmetric design. Other sections have simple patterns of three or four vertical lines.

Portions of two frieze patterns are shown below. Classify the patterns.

a. Following the diagrams on the previous page, you can see that this frieze pattern has rotational symmetry, line symmetry about a horizontal line and a vertical line, and that the pattern can be mapped onto itself by a glide reflection. So, the pattern can be classified as TRHVG.

b. The only transformation that maps this pattern onto itself is a translation. So, the pattern can be classified as T.

EXAMPLE 4 Drawing a Frieze Pattern

TILING A border on a bathroom wall is created using the decorative tile at the right. The border pattern is classified as TR. Draw one such pattern.

SOLUTION

Begin by rotating the given tile 180°. Use this tile and the original tile to create a pattern that has rotational symmetry. Then translate the pattern several times to create the frieze pattern.
1. Describe the term *frieze pattern* in your own words.

2. **ERROR ANALYSIS** Describe Lucy’s error below.

   [Image: This pattern is an example of TR.]

3. In Exercises 3–6, describe the transformations that map the frieze pattern onto itself.

4. 

5. 

6. 

7. List the five possible transformations, along with their letter abbreviations, that can be found in a frieze pattern.

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**Practice and Applications**

**Sweater Pattern** Each row of the sweater is a frieze pattern. Match the row with its classification.

<table>
<thead>
<tr>
<th>A. TRHVG</th>
<th>B. TR</th>
<th>C. TRVG</th>
<th>D. THG</th>
</tr>
</thead>
</table>

8. [Image of sweater pattern]

9. [Image of sweater pattern]

10. [Image of sweater pattern]

11. [Image of sweater pattern]

**Classifying Patterns** Name the isometries that map the frieze pattern onto itself.

12. [Image of sweater pattern]

13. [Image of sweater pattern]

14. [Image of sweater pattern]

15. [Image of sweater pattern]
**DESCRIBING TRANSFORMATIONS** Use the diagram of the frieze pattern.

16. Is there a reflection in a vertical line? If so, describe the reflection(s).
17. Is there a reflection in a horizontal line? If so, describe the reflection(s).
18. Name and describe the transformation that maps A onto F.
19. Name and describe the transformation that maps D onto B.
20. Classify the frieze pattern.

**PET COLLARS** In Exercises 21–23, use the chart on page 438 to classify the frieze pattern on the pet collars.

21.

22.

23.

24. **TECHNOLOGY** Pick one of the seven classifications of patterns and use geometry software to create a frieze pattern of that classification. Print and color your frieze pattern.

25. **DATA COLLECTION** Use a library, magazines, or some other reference source to find examples of frieze patterns. How many of the seven classifications of patterns can you find?

**CREATING A FRIEZE PATTERN** Use the design below to create a frieze pattern with the given classification.

26. TR
27. TV
28. TG
29. THG
30. TRVG
31. TRHVG
The patterns shown were used in Japan during the Tokugawa Shogunate. Classify the frieze patterns.

32. 

33. 

34. 

In Exercises 35–37, use the pottery shown below. This pottery was created by the Acoma Indians. The Acoma pueblo is America’s oldest continually inhabited city.

35. Identify any frieze patterns on the pottery.

36. Classify the frieze pattern(s) you found in Exercise 35.

37. Create your own frieze pattern similar to the patterns shown on the pottery.

38. Look back to the southwestern pottery on page 437. Describe and classify one of the frieze patterns on the pottery.

You are decorating a large circular vase and decide to place a frieze pattern around its base. You want the pattern to consist of ten repetitions of a design. If the diameter of the base is about 9.5 inches, how wide should each design be?

In Exercises 40–42, use the tile to create a border pattern with the given classification. Your border should consist of one row of tiles.

40. TR 

41. TRVG 

42. TRHVG 

Explain how the design of the tiles in Exercises 40–42 is a factor in the classification of the patterns. For instance, could the tile in Exercise 40 be used to create a single row of tiles classified as THG?

Explain why the combination is not a category for frieze pattern classification.
**Student Help**

**Test Preparation**

In Ex. 49–52, use the following information.

In Celtic art and design, border patterns are used quite frequently, especially in jewelry. Three different designs are shown.

A.  
B.  
C.  

49. Use translations to create a frieze pattern of each design.

50. Classify each frieze pattern that you created.

51. Which design does not have rotational symmetry? Use rotations to create a new frieze pattern of this design.

52. **Writing** If a design has 180° rotational symmetry, it cannot be used to create a frieze pattern with classification \( T \). Explain why not.

**Challenge**

The following tree diagram can help classify frieze patterns. Copy the tree diagram and fill in the missing parts.

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**Using the Coordinate Plane** The figure shown in the coordinate plane is part of a frieze pattern with the given classification. Copy the graph and draw the figures needed to complete the pattern.

47. TR  
48. TRVG

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**INTERNET**

**STUDENT HELP**
**MIXED REVIEW**

**RATIOS** Find the ratio of girls to boys in a class, given the number of boys and the total number of students. (Skills Review for 8.1)

54. 12 boys, 23 students
55. 8 boys, 21 students
56. 3 boys, 13 students
57. 19 boys, 35 students
58. 11 boys, 18 students
59. 10 boys, 20 students

**PROPERTIES OF MEDIANS** Given that \( D \) is the centroid of \( \triangle ABC \), find the value of each variable. (Review 5.3)

60.

\[
\begin{align*}
A & \quad 3x - 5 \\
B & \quad 3z \\
C & \quad 7z - 2
\end{align*}
\]

61.

\[
\begin{align*}
A & \quad 3y - 4 \\
B & \quad w - 4 \\
C & \quad 2y
\end{align*}
\]

**FINDING AREA** Find the area of the quadrilateral. (Review 6.7)

62.

\[
\begin{array}{c}
15 \\
12 \\
12 \\
15
\end{array}
\]

63.

\[
\begin{array}{c}
12 \\
20 \\
12 \\
12
\end{array}
\]

64.

\[
\begin{array}{c}
17 \\
18 \\
35 \\
21
\end{array}
\]

**Quiz 2**

Write the coordinates of the vertices \( A', B', \) and \( C' \) after \( \triangle ABC \) is translated by the given vector. (Lesson 7.4)

1. \( \langle 1, 3 \rangle \)
2. \( \langle -3, 4 \rangle \)
3. \( \langle -2, -4 \rangle \)
4. \( \langle 5, 2 \rangle \)

In Exercises 5 and 6, sketch the image of \( \triangle PQR \) after a composition using the given transformations in the order they appear. (Lesson 7.5)

5. \( P(5, 1), Q(3, 4), R(0, 1) \)
   - Translation: \((x, y) \rightarrow (x - 2, y - 4)\)
   - Reflection: in the y-axis

6. \( P(7, 2), Q(3, 1), R(6, -1) \)
   - Translation: \((x, y) \rightarrow (x - 4, y + 3)\)
   - Rotation: 90° clockwise about origin

7. **MUSICAL NOTES** Do the notes shown form a frieze pattern? If so, classify the frieze pattern. (Lesson 7.6)