What is a Quadratic??????

- A quadratic equation is an equation in which the highest power of the variable is a square.
- When graphed, the equation forms a “parabola.”
- The points at which the parabola crosses the x-axis are called the solutions, or roots.

The solutions can be found by looking at the graph.

QUADRATIC FUNCTIONS

The graph of any quadratic function is a parabola. Parabolas have certain common characteristics.

- **Axis of Symmetry**: the line about which the parabola is symmetric; divides a parabola into two mirror images.

- **Vertex**: the point of the parabola where the parabola and axis of symmetry intersect; the highest (or lowest) point of a parabola; the point at which the function has its maximum (or minimum) value.

- The graphs of all parabolas have the same general shape, a U shape.
Section 1:
Identifying the vertex (minimum/maximum), the axis of symmetry, and the roots (zeros):

IDENTIFYING THE VERTEX (MINIMUM/MAXIMUM) AND THE ROOTS (ZEROS):

State the maximum or minimum point (vertex), the axis of symmetry, and the roots (zeros) of the graphs:
Section 2

**Vertex form**  The form $y = a(x - h)^2 + k$, where the vertex of the graph is $(h, k)$ and the axis of symmetry is $x = h$

**Intercept form**  The form $y = a(x - p)(x - q)$, where the $x$-intercepts of the graph are $p$ and $q$

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**Graphing Parabolas**

Using the graphing function of your calculator, place the quadratic into $y= $ then look at the graph. After gaining an understanding of the function, go to the table and plot ALL the points that will fit on the given graph. **Label the vertex and axis of symmetry.**

<table>
<thead>
<tr>
<th>Intercept Form</th>
<th>Standard Form</th>
<th>Vertex Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = (x - 3)(x + 3)$</td>
<td>$y = x^2 - 2x - 4$</td>
<td>$y = -(x - 2)^2 - 3$</td>
</tr>
</tbody>
</table>

**Homework: Worksheet 10.1**
For question 1 - 6, identify the maximum or minimum point, the axis of symmetry, and the roots (zeros) of the graph of the quadratic function shown, as indicated.

Section 1:

1. Maximum point: (___, ___)
   Axis of Symmetry:
   Roots:

2. Minimum point: (___, ___)
   Axis of Symmetry:
   Roots:

3. Minimum point: (___, ___)
   Axis of Symmetry:
   Roots:

4. Maximum point: (___, ___)
   Axis of Symmetry:
   Roots:
Section 2:
For questions 7 - 16, sketch the graph of the function on the provided graphs. Label the vertex and axis of symmetry.

7) \( y = x^2 - 2x - 2 \)

8) \( y = (x+2)(x-2) \)

9) \( y = (x - 3)^2 + 2 \)

10) \( y = -x^2 - 4x - 3 \)
11) \(y = (x-4)(x-2)\)

12) \(y = -(x + 1)^2 - 3\)

13) \(y = x^2 + 4x + 3\)

14) \(y = (x-1)(x-3)\)

15) \(y = -(x - 2)^2 - 2\)

16) \(y = -x^2 + 4x + 1\)
Section 1: Solving Quadratic Equations by Graphing

Solve the quadratic equation by graphing:
\[ x^2 + x - 6 = 0 \]

![Graph of \( x^2 + x - 6 = 0 \)]

Solutions are __________________

Directions for graphing using a graphing calculator:
Place the function into the "y=" function on the calculator. Press "Graph" to see where the graph crosses the x-axis. Press "2nd" then "Graph" to see the list of ordered pairs for the graph. On your paper, plot all ordered pairs from that list that will fit on your graph. The \( x\)-value of the ordered pair where the graph crosses (or touches) the x-axis are the solutions (Zeros) to the quadratic equation.

Section 2

First, make sure the equation is equal to zero.
Second, factor the equation.
Third, set each factor equal to zero and solve for \( x \).
( Remember Unit 9 Lesson 1???

Solve the equation by factoring:
\[ 3x^2 - 8x = -5 \]

Solve the equation by factoring:
\[ 5x^2 + 13x = 6 \]
Section 3

We use this method when the equation has a quantity squared in it such as $3(x - 3)^2 + 2 = 38$.

**Extracting Square Roots:**
Solve by extracting the square roots. Leave answer in simplest radical form.

$$3(x - 3)^2 + 2 = 38$$

**Steps:**
- **First**, get the squared binomial, $(x - 3)^2$, by itself on one side of the equal sign. (Follow the rules for solving equations treating the quantity $(x - 3)^2$ like a single variable.
- **Second**, take the square root of both sides. Don’t forget to:
  - put the $\pm$ sign when you take the square root
  - simplify the square root, if possible
- **Third**, get $x$ by itself by adding or subtracting.
- **Fourth**, be sure to write your answer in the form of
  $$x = 3 \pm 2\sqrt{3}$$

Solve by extracting the square roots. Leave answer in simplest radical form.

$$2(x - 2)^2 + 4 = 20$$

**Steps:**
- **First**, get the squared binomial, by itself on one side of the equal sign.
- **Second**, take the square root of both sides. Don’t forget to:
  - put the $\pm$ sign when you take the square root
  - simplify the square root, if possible
- **Third**, get $x$ by itself by adding or subtracting.
- **Fourth**, be sure to write your answer in the proper form!
Section 4
Solve the equation by completing the square. Leave answer in simplest radical form:

\[ x^2 + 6x - 7 = 0 \]

<table>
<thead>
<tr>
<th>This is the original equation.</th>
<th>[ x^2 + 6x - 7 = 0 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Move the loose number over to the other side.</td>
<td>[ x^2 + 6x = 7 ]</td>
</tr>
<tr>
<td>Take half of the ( x )-term (that is, divide it by two) (and don't forget the sign!), and square it. Add this square to both sides of the equation.</td>
<td>[ x^2 + 6x + 9 = 7 + 9 ]</td>
</tr>
<tr>
<td>Convert the left-hand side to squared form. Simplify the right-hand side.</td>
<td>[ (x + 3)^2 = 16 ]</td>
</tr>
<tr>
<td>Square-root both sides. Remember to do &quot;( \pm )&quot; on the right-hand side.</td>
<td>[ x + 3 = \pm 4 ]</td>
</tr>
<tr>
<td>Solve for ( x = ). Remember that the &quot;( \pm )&quot; gives you two solutions. Simplify as necessary.</td>
<td>[ x = -3 \pm 4 ]</td>
</tr>
<tr>
<td>[ = -3 - 4, -3 + 4 ]</td>
<td></td>
</tr>
<tr>
<td>[ = -7, 1 ]</td>
<td></td>
</tr>
</tbody>
</table>

Solve the equation by completing the square. Leave answer in simplest radical form:
\[ x^2 + 6x - 12 = 0 \]

Solve the equation by completing the square. Leave answer in simplest radical form:
\[ x^2 + 4x = 18 \]
Section 5
Generally, we use the quadratic formula to find the solutions when we are unable to find them by factoring or graphing (decimal answers). But to get started, let’s see what this will look like on one we would know the answer to already.

- Solve $x^2 + 3x - 4 = 0$

This quadratic happens to factor:

$$x^2 + 3x - 4 = (x + 4)(x - 1) = 0$$

...so I already know that the solutions are $x = -4$ and $x = 1$. How would my solution look in the Quadratic Formula? Using $a = 1$, $b = 3$, and $c = -4$, my solution looks like this:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{9 + 16}}{2} = \frac{-3 \pm \sqrt{25}}{2} = \frac{-3 \pm 5}{2}$$

$$x = \frac{-8}{2}, \quad \frac{2}{2} = -4, \quad 1 = x$$

Then, as expected, the solution is $x = -4, x = 1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the equation by using the quadratic formula.

- $x^2 + x - 6 = 0$
- $x^2 - 3x + 4 = 0$

Homework:
Worksheet 10.2
Solving Quadratic Equations

Section 1 (Graphing):
Solve the equation by graphing:
(Must show graphing)
1. \( x^2 - 2x - 8 = 0 \)
2. \( x^2 + 6x + 5 = 0 \)
3. \( x^2 - 16 = 0 \)
4. \( -x^2 + 7x - 6 = 0 \)

Section 2 (Factoring):
Solve the equation by factoring:
(Must show factoring)
5. \( 3x^2 - 13x = -4 \)
6. \( 2x^2 + 3x = 9 \)
Section 3 (Extracting square roots):

Solve the equation by extracting the square roots. Leave answers in simplest radical form.
(Must show extracting square roots.)

9. \(2(x-4)^2 + 1 = 17\)

10. \(2(x-2)^2 + 5 = 21\)

11. \(3(x-3)^2 + 3 = 27\)

12. \(4(x-5)^2 + 2 = 34\)
Section 4 (Completing The Square):

Solve the equation by completing the square. Leave answers in simplest radical form.
(Must show completing the square)

13. \( x^2 + 2x = 9 \)

14. \( x^2 + 6x = 12 \)

15. \( x^2 - 2x = 14 \)

16. \( x^2 - 6x = 9 \)
Section 5 (Quadratic Formula):

Solve the equation by using the quadratic formula.

17. \( x^2 - 11x + 28 = 0 \)

18. \( x^2 + 8x + 12 = 0 \)

19. \( x^2 - 2x - 8 = 0 \)

20. \( x^2 + 6x + 5 = 0 \)
10.3 Notes
Writing Quadratic Equations

Section 1
STANDARD Form: \( y = ax^2 + bx + c \)
(Multiply the binomials)

Write the given quadratic function in standard form:
\( y = (x - 2)(x + 3) \)

Write the given quadratic function in standard form:
\( y = (2x + 1)(x - 4) \)

CHECKING YOUR ANSWER:
A good way to check you answer, is to plug in the original function in your calculator and graph it. Now, plug in your answer and graph it on the same screen. If you only see one function, then you wrote the function correctly, because they are the same function, just written differently. ☺
Section 2

- First, you need to write the roots in intercept form.

**Intercept Form:** \( y = (x - p)(x - q) \), where \( p \) is your first root and \( q \) is your second root.
- Second, multiply out the binomials like in section 1.

Write the quadratic function in standard form given the roots:
-2 and 3

Write the quadratic function in standard form given the roots:
0 and 6

Section 3

**VERTEX Form:** \( y = (x - h)^2 + k \), where \( h \) is the \( x \)-value of the vertex and \( k \) is the \( y \)-value of the vertex.

In order to get the standard form on the quadratic into vertex form, we can complete the square like in lesson 10.2 or find the vertex and plug into vertex form.

Write the given quadratic function in vertex form: \( y = x^2 - 4x + 8 \)

Write the given quadratic function in vertex form: \( y = x^2 + 10x + 17 \)
Section 4

- In order to find the vertex, we have to find the axis of symmetry first; \( x = -\frac{b}{2a} \)
- Remember, we get the \( a \) and \( b \) from the function.
- Once we find the \( x \)-value (axis of symmetry), we plug it in and find the \( y \)-value.

Find the minimum (vertex) algebraically of the equation.
\[ y = 2x^2 + 12x + 13 \]

Find the maximum (vertex) algebraically of the equation.
\[ y = -x^2 + 2x - 4 \]

Section 5

INTERCEPT Form: \( y = (x - p)(x - q) \)

In order to get the standard form on the quadratic into intercept form, we have to factor the trinomial.

Write the quadratic function in intercept form.
\[ y = x^2 - 11x + 18 \]

Find the minimum (vertex) algebraically of the equation.
\[ y = x^2 + 7x - 8 \]

Homework: Worksheet 10.3
Writing Quadratic Equations

Section 1 (Standard Form):
Write the given quadratic functions in standard form:

1. \( y = (x - 2)(x + 4) \)

2. \( y = (x + 1)(2x + 5) \)

3. \( y = (x - 4)(x + 4) \)

4. \( y = -(x - 1)(x - 6) \)

5. \( y = -(3x + 2)(x + 1) \)

Section 2 (Standard Form Given Roots):
Write the quadratic functions in standard form given the roots.

6. \(-4\) and \(-1\)
Section 3 (Vertex Form):
Write the given quadratic functions in vertex form.

11. \( y = x^2 + 2x + 4 \)

12. \( y = x^2 - 8x + 14 \)

13. \( y = x^2 + 6x - 1 \)
14. \( y = x^2 - 14x + 50 \)

15. \( y = x^2 + 4x - 2 \)

16. \( y = x^2 - 2x - 1 \)

17. \( y = -x^2 + 4x + 5 \)

18. \( y = 3x^2 - 12x - 8 \)

19. \( y = 2x^2 + 8x + 1 \)

20. \( y = x^2 - x + 3 \)

**Section 4 (Find the Vertex):**

Find the minimum or maximum (vertex) algebraically of the following equations. Show all work.

16. \( y = x^2 - 2x - 1 \)
Section 5 (Intercept Form):

Write the quadratic functions in intercept form.

21. \( y = x^2 - 11x + 28 \)

22. \( y = x^2 + 8x + 12 \)

23. \( y = x^2 - 25 \)

24. \( y = 2x^2 - 3x - 9 \)

25. \( y = 3x^2 - 13x + 4 \)