The well-known right triangle relationship called the Pythagorean Theorem is named for Pythagoras, a Greek mathematician who lived in the sixth century B.C. We now know that the Babylonians, Egyptians, and Chinese were aware of this relationship before its discovery by Pythagoras.

There are many proofs of the Pythagorean Theorem. You will see one proof in Exercise 48 and others later in the book.

A Pythagorean triple is a set of nonzero whole numbers $a$, $b$, and $c$ that satisfy the equation $a^2 + b^2 = c^2$. Here are some common Pythagorean triples.

- $3, 4, 5$
- $5, 12, 13$
- $8, 15, 17$
- $7, 24, 25$

If you multiply each number in a Pythagorean triple by the same nonzero whole number, the three numbers that result also form a Pythagorean triple.

**Key Concepts**

**Theorem 8-1**

*Pythagorean Theorem*

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$

A **Pythagorean triple** is a set of nonzero whole numbers $a$, $b$, and $c$ that satisfy the equation $a^2 + b^2 = c^2$. Here are some common Pythagorean triples.

- $3, 4, 5$
- $5, 12, 13$
- $8, 15, 17$
- $7, 24, 25$

If you multiply each number in a Pythagorean triple by the same nonzero whole number, the three numbers that result also form a Pythagorean triple.

**Vocabulary Tip**

- **Hypotenuse**
- **Legs**

**California Content Standards**

**GEOM 15.0** Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles. Introduce

**California Math Background**

Although several ancient cultures postulated the Pythagorean Theorem and used it to measure distances, the first proof of it was attributed by Euclid to Pythagoras. The distance formula is a coordinate form of the Pythagorean Theorem, which is the foundation of all trigonometric functions.

**More Math Background:** p. 414C

**Lesson Planning and Resources**

See p. 414E for a list of the resources that support this lesson.
2. Teach

Guided Instruction

1. **Pythagorean Triples**
   - **Teaching Tip**
     Let students know that Pythagorean triples often appear on standardized tests.
   - **Error Prevention**
     Some students may assume that the legs are always the known quantities. Point out that c is always the hypotenuse when applying the formula \(a^2 + b^2 = c^2\) to a right triangle.
   - **Technology Tip**
     Students may wonder why they are asked to use a calculator in some exercises but not in other similar exercises. Tell them that real-world applications typically require decimal answers. Point out that radicals are exact, so they are preferred when exercises are of a purely mathematical nature.

2. **Using Simplest Radical Form**
   - **Algebra**
     Find the value of \(x\). Leave your answer in simplest radical form (page 390).
     
     \[
     \begin{align*}
     a^2 + b^2 &= c^2 \\
     21^2 + 20^2 &= c^2 \\
     441 + 400 &= c^2 \\
     841 &= c^2 \\
     c &= 29
     \end{align*}
     
     Take the square root.
     
     The length of the hypotenuse is 29. The lengths of the sides 20, 21, and 29 form a Pythagorean triple because they are whole numbers that satisfy \(a^2 + b^2 = c^2\).

3. **Real-World Connection**
   - **Technology Tip**
     If students have trouble seeing the connection between the Pythagorean Theorem and the real world, draw a baseball diamond on the board. Explain that a baseball field is a square with 90-foot sides. The home plate and second base are at opposite vertices of the square. Illustrate how to find the distance between them using the Pythagorean Theorem.

4. **Critical Thinking**
   - When you want to know how far you have to paddle a boat, why is an approximate answer more useful than an answer in simplest radical form?
   - It is 430 m from one dock to the other.

Additional Examples

1. A right triangle has legs of length 16 and 30. Find the length of the hypotenuse. Do the lengths of the sides form a Pythagorean triple? \(\text{yes}\)

2. Find the value of \(x\). Leave your answer in simplest radical form.

3. A baseball diamond is a square with 90-ft sides. Home plate and second base are at opposite vertices of the square. About how far is home plate from second base? \(\text{about 127 ft}\)

4. The Parks Department rents paddle boats at docks near each entrance to the park. To the nearest meter, how far is it to paddle from one dock to the other?

   \[
   \begin{align*}
   a^2 + b^2 &= c^2 \\
   250^2 + 350^2 &= c^2 \\
   62500 + 122500 &= c^2 \\
   185000 &= c^2 \\
   c &= \sqrt{185000} \\
   c &= 430.11626
   \end{align*}
   
   Use a calculator.

   It is 430 m from one dock to the other.
Lesson 8-1 The Pythagorean Theorem and Its Converse

You can use the Converse of the Pythagorean Theorem to determine whether a triangle is a right triangle. You will prove Theorem 8-2 in Exercise 58.

Is It a Right Triangle?

Is this triangle a right triangle?

\[ c^2 = a^2 + b^2 \]

Substitute the greatest length for \( c \).

7225

Simplify.

7225

✓

\[ c^2 = a^2 + b^2, \text{ so the triangle is a right triangle.} \]

A triangle has sides of lengths 16, 48, and 50. Is the triangle a right triangle?

You can also use the squares of the lengths of the sides of a triangle to find whether the triangle is acute or obtuse. The following two theorems tell how.

Classifying Triangles as Acute, Obtuse, or Right

Classify the triangle whose side lengths are 6, 11, and 14 as acute, obtuse, or right.

\[ 14^2 \geq 6^2 + 11^2 \]

\[ 196 \geq 36 + 121 \]

\[ 196 > 157 \]

Since \( c^2 > a^2 + b^2 \), the triangle is obtuse.

A triangle has sides of lengths 7, 8, and 9. Classify the triangle by its angles.

Acute

Guided Instruction

Technology Tip

Have students use geometry software to explore and demonstrate the theorems. If \( c^2 > a^2 + b^2 \), the triangle is obtuse and if \( c^2 < a^2 + b^2 \), the triangle is acute. Direct students to keep \( a \) and \( b \) constant while manipulating \( c \) by altering the angle opposite \( c \).

Example Error Prevention

Remind students that \( c \) must be the longest side of the triangle for the comparison of \( c^2 \) and \( a^2 + b^2 \) to give a valid triangle classification. Also, students should use the Triangle Inequality Theorem to check that \( a + b > c \) so that the side lengths form a triangle.

Example Additional Examples

Is this triangle a right triangle?

no

The numbers represent the lengths of the sides of a triangle. Classify each triangle as acute, obtuse, or right.

a. 15, 20, 25 right
b. 10, 15, 20 obtuse

Resources

• Daily Notetaking Guide 8-1
• Daily Notetaking Guide 8-1—Adapted Version

Additional Examples

Is this triangle a right triangle?

no

The numbers represent the lengths of the sides of a triangle. Classify each triangle as acute, obtuse, or right.

a. 15, 20, 25 right
b. 10, 15, 20 obtuse

Resources

• Daily Notetaking Guide 8-1
• Daily Notetaking Guide 8-1—Adapted Version

Closure

The area of \( \triangle ABC \) is 20 ft\(^2\). Find \( AC \) and \( BC \). Leave your answer in simplest radical form.

\[ AC = 2\sqrt{5} \text{ ft}; BC = 4\sqrt{5} \text{ ft} \]
### 3. Practice

**Assignment Guide**


**2. A B** 18–26, 30, 31, 33, 40–47

**C Challenge** 54–58

Multiple Choice Practice 59, 60

Mixed Review 61–69

**Homework Quick Check**

To check students’ understanding of key skills and concepts, go over Exercises 16, 18, 30, 36, 50.

**Exercises 14, 15** These exercises anticipate the special right triangle relationships in Lesson 8–2. Ask: What is the ratio a : b : c in each triangle? 1 : 1 : \( \sqrt{2} \)

**Exercises 21–26** In only some of the exercises do the first two lengths represent a and b. Remind students to compare the sum of the squares of the two smaller lengths with the square of the greatest length.

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### EXERCISES

**Standards Practice**

For more exercises, see Extra Skill, Word Problem, and Proof Practice.

### Practice by Example

**Example 1** (page 418)

Find the value of each variable. Leave your answers in simplest radical form.

1. 2. 3. 4.

**Example 2** (page 418)

Find the value of \( x \). Leave your answer in simplest radical form.

7. 8. 9. 10.

**Example 3** (page 418)


**Example 4** (page 419)

15. 16. 17.

**Example 5** (page 419)

18. 19. 20.

---

**Does each set of numbers form a Pythagorean triple? Explain.**

7. 4, 5, 6

8. 10, 24, 26

9. 15, 20, 25

**Example 2** (page 418)

Find the value of \( x \). Leave your answer in simplest radical form.

10. \( \sqrt{41} \)

11. \( \sqrt{33} \)

12. \( 3\sqrt{11} \)

13. \( 2\sqrt{89} \)

14. \( 3\sqrt{2} \)

15. \( 5\sqrt{2} \)

**Example 3** (page 418)

16. **Home Maintenance** A painter leans a 15-ft ladder against a house. The base of the ladder is 5 ft from the house.

   a. To the nearest tenth of a foot, how high on the house does the ladder reach? 14.1 ft

   b. The ladder in part (a) reaches too high on the house. By how much should the painter move the ladder’s base away from the house to lower the top by 1 ft? about 2.3 ft

17. A walkway forms the diagonal of a square playground. The walkway is 24 m long. To the nearest tenth of a meter, how long is a side of the playground? 17.0 m

**Example 4** (page 419)

18. 19. 20.

**Example 5** (page 419)

21. \( 4, 5, 6 \) acute

22. \( 0.3, 0.4, 0.6 \) obtuse

23. \( 11, 12, 15 \) acute

24. \( \sqrt{3}, 2, 3 \) obtuse

25. 30, 40, 50 right

26. \( \sqrt{11}, \sqrt{7}, 4 \) acute


30. **Answers may vary.**

Sample: Have three people hold the rope 3 units, 4 units, and 5 units apart in the shape of a triangle.

31. **Multiple Choice** Which triangle is not a right triangle? **B**

A. 

B. 

C. 

D. 

32. **Embroidery** You want to embroider a square design. You have an embroidery hoop with a 6 in. diameter. Find the largest value of $x$ so that the entire square will fit in the hoop. Round to the nearest tenth. 4.2 in.


Yes, $7^2 + 24^2 = 25^2$, so $\triangle RST$ is a rt. 

34. **Coordinate Geometry** You can use the Pythagorean Theorem to prove the Distance Formula. Let points $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the endpoints of the hypotenuse of a right triangle.

a. Write an algebraic expression to complete each of the following:

$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

b. By the Pythagorean Theorem, $PQ = PR + QR$. Rewrite this statement substituting the algebraic expressions you found for $PR$ and $QR$ in part (a).

c. Complete the proof by taking the square root of each side of the equation that you wrote in part (b).

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

35. **Constructions** Explain how to construct a segment of length $\sqrt{n}$, where $n$ is any positive integer, and you are given a segment of length 1. (Hint: See the diagram.) **See margin.**

Find a third whole number so that the three numbers form a Pythagorean triple.

36. 20, 21

37. 14, 48

38. 13, 85

39. 12, 37

40. Answers may vary.

Sample: Using 2 segments of length 1, construct the hyp. of the right $\triangle$ formed by these segments. Using the hyp. found as one leg and a segment of length 1 as the other leg, construct the hyp. of the $\triangle$ formed by those legs. Continue this process until constructing a hypotenuse of length $\sqrt{n}$. **See margin.**
4. Assess & Reteach

**48. Prove the Pythagorean Theorem.**

**Given:** \( \triangle ABC \) is a right triangle

**Prove:** \( a^2 + b^2 = c^2 \)

(Hint: Begin with proportions suggested by Theorem 7-3 and its corollaries.)

**49. Astronomy** The Hubble Space Telescope is orbiting Earth 600 km above Earth’s surface. Earth’s radius is about 6370 km. Use the Pythagorean Theorem to find the distance \( x \) from the telescope to Earth’s horizon. Round your answer to the nearest ten kilometers.

2830 km

The figures below are drawn on centimeter grid paper.

Find the perimeter of each shaded figure to the nearest tenth.

**45. 14; 16**

**46. 18; 19**

**47. 39; 42**

**51. 12 cm**

**52. 12.5 cm**

**53. a.** The ancient Greek philosopher Plato used the expressions \( 2n^2 + 1 \), and \( n^2 + 1 \) to produce Pythagorean triples. Choose any integer greater than 1. Substitute for \( n \) and evaluate the three expressions.

\[ 12^2 + 35^2 = 37^2 \]

**b.** Verify that your answers to part (a) form a Pythagorean triple.

**c.** Show that, in general, \( (2n)^2 + (n^2 - 1)^2 = (n^2 + 1)^2 \) for any \( n \). See left.

**54. Geometry in 3 Dimensions** The box at the right is a rectangular solid.

- **a.** Use \( \triangle ABC \) to find the length \( d_1 \) of the diagonal of the base. 5 in.
- **b.** Use \( \triangle ABD \) to find the length \( d_2 \) of the diagonal of the box. \( \sqrt{29} \)
- **c.** You can generalize the steps in parts (a) and (b).

Use the facts that \( AC^2 + BC^2 = d_1^2 \) and \( AD^2 + BD^2 = d_2^2 \) to write a one-step formula to find \( d_3 \).

- **d.** Use the formula you wrote to find the length of the longest fishing pole you can pack in a box with dimensions 18 in., 24 in., and 16 in. 34 in.

**Geometry in 3 Dimensions** Points \( P(x_1, y_1, z_1) \) and \( Q(x_2, y_2, z_2) \) at the left are points in a three-dimensional coordinate system. Use the following formula to find \( PQ \). Leave your answer in simplest radical form.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \]

**55.** \( P(0, 0, 0), Q(1, 2, 3) \)

**56.** \( P(0, 0, 0), Q(-3, 4, -6) \)

**57.** \( P(-1, 3, 5), Q(2, 1, 7) \)

**58.** \( \sqrt{14} \)

**59.** \( \sqrt{61} \)

**60.** \( \sqrt{17} \)
58. Use the plan and write a proof of Theorem 8-2, the Converse of the Pythagorean Theorem.

**Given:** \( \triangle ABC \) with sides of length \( a \), \( b \), and \( c \) where \( a^2 + b^2 = c^2 \)

**Prove:** \( \triangle ABC \) is a right triangle.

**Plan:** Draw a right triangle (not \( \triangle ABC \)) with legs of lengths \( a \) and \( b \). Label the hypotenuse \( c \). By the Pythagorean Theorem, \( a^2 + b^2 = c^2 \). Use substitution to compare the lengths of the sides of your triangle and \( \triangle ABC \). Then prove the triangles congruent. **See margin.**

**Multiple Choice Practice**

For California Standards and CAHSEE Tutorials, visit PHSchool.com. Web Code: bcq-9045

**GEOM 15.0** 59. A highway detour affects a company’s delivery route. The plan showing the old route and the detour is at the right. How many extra miles will the trucks travel once the detour is established? B

- \( \Box \) 6.5
- \( \Box \) 9.1
- \( \Box \) 2.6
- \( \Box \) 1.3

**GEOM 12.0** 60. Determine the value of \( x \) in the figure at the right. B

- \( \Box \) 8
- \( \Box \) 18
- \( \Box \) 9
- \( \Box \) 20

**Mixed Review**

**Lesson 4-7**

For the figure at the right, complete the proportion.

61. \( \frac{10}{15} = \frac{y}{18} \)
62. \( \frac{x}{y} = \frac{\overline{ST}}{\overline{RS}} \)
63. Find the values of \( x \) and \( y \). \( x = 21, y = 12 \)

**Lesson 5-2**

In the second figure, \( \overline{FR} \) bisects \( \angle RPE \). Solve for each variable. Then find \( RS \).

- \( \Box \) 7: 33
- \( \Box \) 3: 20

**Lesson 4-1**

\( \angle PQR \equiv \angle STV \). Solve for each variable.

- \( \Box \) 10
- \( \Box \) 6
- \( \Box \) 6
- \( \Box \) 7

**Exercise 58** Challenge students to prove the Converse of the Pythagorean Theorem using coordinate methods.

Given: \( a^2 + b^2 = c^2 \)

Prove (using slopes):

- \( \triangle ABC \) with legs \( DE \), \( DF \), hyp. \( EF \) of length \( a \), \( b \), and hyp. \( \overline{EF} \) of length \( \overline{c} \). Then \( a^2 + b^2 = c^2 \).

Proof: \( a^2 = m^2 + n^2 \);

\( b^2 = (c - m)^2 + n^2 \);

\( c^2 = a^2 + b^2 = m^2 + n^2 + (c - m)^2 + n^2 = m^2 + n^2 + m^2 - 2mc + c^2 + n^2 \), so \( 0 = 2m^2 + 2n^2 - 2mc \), which simplifies to \( n^2 = mc - m^2 \).