3 TRANSLATIONS, REFLECTIONS, AND ROTATIONS

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To begin this chapter, cut out the figures shown on this page. You will have a trapezoid, two triangles, and a parallelogram. You will be using these figures in several lessons. What do you know about these shapes?
Problems 1 Sliding to the Right, Left, Up, and Down

Let’s explore different ways to move, or transform, figures across a coordinate plane. A transformation is the mapping, or movement, of all the points of a figure in a plane according to a common operation.

1. Look at the parallelogram shown on the coordinate plane.

a. Place your parallelogram on the original figure on the coordinate plane shown and slide it 5 units to the left. Trace your parallelogram on the coordinate plane, and label it Figure 1.

b. Place your parallelogram on the original figure on the coordinate plane shown and slide it 5 units down. Trace your parallelogram on the coordinate plane, and label it Figure 2.

c. Place your parallelogram on Figure 1 on the coordinate plane and slide it 5 units down. Trace your parallelogram on the coordinate plane, and label it Figure 3.

d. Describe how all of the parallelograms you traced on the coordinate plane compare with each other.
2. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
   a. Did sliding the parallelogram either up or down on the coordinate plane change the size or shape of the parallelogram?
   b. Are Figure 1, Figure 2, and Figure 3 all congruent to the original parallelogram shown on the coordinate plane? Explain your reasoning.

When you were sliding the parallelogram to the different places, you were performing translations of the parallelogram. A translation is a transformation that "slides" each point of a figure the same distance and direction. Sliding a figure left or right is a horizontal translation, and sliding it up or down is a vertical translation. The new figure created from the translation is called the image. The original figure is called the pre-image.

3. Look at the triangle shown on the coordinate plane.

   a. List the ordered pairs for the vertices of $\triangle ABC$.
b. Place your triangle on $\triangle ABC$, and translate it 6 units vertically. Trace the new triangle, and label the vertices $A'$, $B'$, and $C'$ in $\triangle A'B'C'$ so the vertices correspond to the vertices $A$, $B$, and $C$ in $\triangle ABC$.

c. List the ordered pairs for the vertices of $\triangle A'B'C'$.

d. Place your triangle on $\triangle ABC$, and translate it 6 units horizontally. Trace the new triangle, and label the vertices $A''$, $B''$, and $C''$ in $\triangle A''B''C''$ so the vertices correspond to the vertices $A$, $B$, and $C$ in $\triangle ABC$.

e. List the ordered pairs for the vertices of $\triangle A''B''C''$.

f. Compare the ordered pairs in $\triangle ABC$ and $\triangle A'B'C'$. How are the values in the ordered pairs affected by the translation?

g. Compare the ordered pairs in $\triangle ABC$ and $\triangle A''B''C''$. How are the values in the ordered pairs affected by the translation?

h. If you were to translate $\triangle ABC$ 10 units vertically to form $\triangle DEF$, what would be the ordered pairs of the corresponding vertices?

i. If you were to translate $\triangle ABC$ 10 units horizontally to form $\triangle GHJ$, what would be the ordered pairs of the corresponding vertices?
4. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
   a. Did sliding the triangle either up or down on the coordinate plane change the size or shape of the triangle?
   
   b. Are both of the triangles you drew congruent to the triangle shown on the coordinate plane? Explain your reasoning.

5. Look at the triangle shown on the coordinate plane.

   a. List the ordered pairs for the vertices of \( \triangle ABC \).
   
   b. Place your triangle on \( \triangle ABC \), and translate it \(-5\) units vertically. Trace the new triangle, and label the vertices \( A' \), \( B' \), and \( C' \) in \( \triangle A'B'C' \) so the vertices correspond to the vertices \( A, B, \) and \( C \) in \( \triangle ABC \).
   
   c. List the ordered pairs for the vertices of \( \triangle A'B'C' \).
d. Place your triangle on \( \triangle ABC \), and translate it –5 units horizontally. Trace the new triangle, and label the vertices \( A'^{\prime} \), \( B'^{\prime} \), and \( C'^{\prime} \) in \( \triangle A'B'C' \) so the vertices correspond to the vertices \( A, B, \) and \( C \) in \( \triangle ABC \).

e. List the ordered pairs for the vertices of \( \triangle A'B'C' \).

f. Compare the ordered pairs in \( \triangle ABC \) and \( \triangle A'B'C' \). How are the values in the ordered pairs affected by the translation?

g. Compare the ordered pairs in \( \triangle ABC \) and \( \triangle A'B'C' \). How are the values in the ordered pairs affected by the translation?

h. If you were to translate \( \triangle ABC \) 10 units vertically to form \( \triangle DEF \), what would be the ordered pairs of the corresponding vertices?

i. If you were to translate \( \triangle ABC \) 10 units horizontally to form \( \triangle GHJ \), what would be the ordered pairs of the corresponding vertices?

6. Are both triangles congruent to the original triangle shown on the coordinate plane? Explain your reasoning.
Problem 2  Translating a Trapezoid

1. Look at the trapezoid shown on the coordinate plane.

a. List the ordered pairs for the vertices of trapezoid $ABCD$.

b. Place your trapezoid on trapezoid $ABCD$, and translate it $-5$ units vertically. Trace the new trapezoid, and label the vertices $A'$, $B'$, $C'$, and $D'$ in trapezoid $A'B'C'D'$ so the vertices correspond to the vertices $A$, $B$, $C$, and $D$ in trapezoid $ABCD$.

c. List the ordered pairs for the vertices of trapezoid $A'B'C'D'$.

d. Place your trapezoid on trapezoid $ABCD$, and translate it $-5$ units horizontally. Trace the new trapezoid, and label the vertices $A''$, $B''$, $C''$, and $D''$ in trapezoid $A''B''C''D''$ so the vertices correspond to the vertices $A$, $B$, $C$, and $D$ in trapezoid $ABCD$.

e. List the ordered pairs for the vertices of trapezoid $A''B''C''D''$.

Can you predict what will happen to the ordered pairs of the trapezoid?
f. Compare the ordered pairs in trapezoid \(ABCD\) and trapezoid \(A'B'C'D'\). How are the values in the ordered pairs affected by the translation?

g. Compare the ordered pairs in trapezoid \(ABCD\) and trapezoid \(A''B''C''D''\). How are the values in the ordered pairs affected by the translation?

h. If you were to translate trapezoid \(ABCD\) 10 units vertically to form trapezoid \(DEFG\), what would be the ordered pairs of the corresponding vertices?

i. If you were to translate trapezoid \(ABCD\) 10 units horizontally to form trapezoid \(HJKM\), what would be the ordered pairs of the corresponding vertices?

2. Recall that two geometric figures are considered congruent when they are the same size and the same shape.

a. Did sliding the trapezoid either up or down on the coordinate plane change the size or shape of the trapezoid?

b. Are both trapezoids congruent to the original trapezoid shown on the coordinate plane? Explain your reasoning.
Talk the Talk

1. Are all images, or new figures that result from a translation, always congruent to the original figure? Explain your reasoning.

2. For any real number \( c \) or \( d \), describe how the ordered pair \((x, y)\) of any original figure will change when translated:

   a. horizontally \( c \) units. How do you know if the image translated to the left or to the right?

   b. vertically \( d \) units. How do you know if the image translated up or down?

Be prepared to share your solutions and methods.
Rotations of Geometric Figures on the Coordinate Plane

Learning Goal
In this lesson, you will:
- Rotate geometric figures on the coordinate plane.

Key Terms
- rotation
- angle of rotation
- point of rotation

Centrifuges are devices that spin material around a center point. Centrifuges are used in biology and chemistry, often to separate materials in a gas or liquid.

Tubes are inserted into the device and, as it spins, heavier material is pushed to the bottom of the tubes while lighter material tends to rise to the top.

Human centrifuges are used to test pilots and astronauts. Can you think of other devices that work like centrifuges?
Problem 1  What Is a Rotation?

You have considered what happens to shapes when you slide them up, down, left, or right. Let’s explore what happens when you rotate a geometric figure.

1. Look at the triangles shown in the coordinate plane.

2. Place your triangle on $\triangle ABC$. Without moving vertex $A$, “transform” the triangle into $\triangle AB'C'$.

3. Describe how you transformed the triangle.

4. Katie says that she can use translations to move triangle $ABC$ to triangle $AB'C'$. Is she correct? Explain your reasoning.
A rotation is a transformation that turns a figure about a fixed point for a given angle, called the angle of rotation, and a given direction. The angle of rotation is the amount of rotation about a fixed point, or point of rotation. Rotation can be clockwise or counterclockwise.

The point of rotation can be a point on the figure.

Or, it can be a point not on the figure.

It can also be a point in the figure.
5. Use your triangle to rotate $\triangle ABC$ in Question 1 by placing your triangle on the figure, putting a pin in it at vertex $C$, and then rotating your triangle first to the left and then to the right.

6. Using $\overline{AC}$ as one side of the angle, measure and draw $\angle ACA'$ to be $120^\circ$. Then, rotate your triangle clockwise to produce $\triangle CA'B'$. Label your rotation in the coordinate plane.

7. Use your triangle to rotate $\triangle ABC$ by placing your triangle on the figure, putting a pin in it at any point on side $\overline{AC}$, and then rotating your triangle first clockwise and then counterclockwise. Trace one rotation you performed on the coordinate plane as $\triangle A''B''C''$. You will need your protractor.

Place a point at your point of rotation.

You will need your protractor.
8. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
   a. Did rotating the triangle on the coordinate plane in any of the previous questions change the size or shape of the triangle?
   b. Is the image of the triangle that resulted from the rotation congruent to the triangle shown on the coordinate plane? Explain your reasoning.

**Problem 2  Rotating a Parallelogram**

1. Use your parallelogram to rotate parallelogram $ABCD$ by placing your parallelogram on the figure, putting a pin in it at any point in the interior of the parallelogram, and then rotating your parallelogram first clockwise and then counterclockwise. Trace one rotation you performed on the coordinate plane. Place a point at your center of rotation.
2. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
   a. Did rotating the parallelogram on the coordinate plane change the size or shape of the parallelogram?
   b. Is the image of the parallelogram that resulted from the rotation congruent to the parallelogram shown on the coordinate plane? Explain your reasoning.

Problem 3 Rotating a Trapezoid

1. Use your trapezoid to rotate trapezoid $ABCD$ around point $P$ by placing your trapezoid on the figure. Fold a piece of tape in half and tape it to both sides of the trapezoid, making sure that the tape covers point $P$. Put a pin in at point $P$, and rotate your parallelogram first clockwise and then counterclockwise. Trace one rotation you performed on the coordinate plane.

Don't tape your trapezoid to your paper!
2. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
   a. Did rotating the trapezoid on the coordinate plane change the size or shape of the trapezoid?
   b. Is the image of the trapezoid congruent to the trapezoid shown on the coordinate plane? Explain your reasoning.

Talk the Talk

1. Are all images, or new figures that result from a rotation, always congruent to the original figure? Explain your reasoning.
2. Describe the point of rotation in each.

a.

b.

So is the point of rotation in the figure, on the figure, or not on the figure?

c.

Be prepared to share your solutions and methods.
The astronauts aboard the Apollo Moon missions in 1969 through the 1970s did more than just play golf and take pictures. They also set up equipment on the Moon to help scientists measure the distance from the Moon to the Earth. This equipment contained sets of mirrors, called retroreflectors. Scientists on Earth can now shoot laser beams at these mirrors and calculate the distance to the Moon by observing how long it takes the laser beam to "bounce back."
Problem 1  Reflections Across the Axes

In this lesson you will explore what happens to geometric figures that are reflected across different lines.

1. Look at the two triangles shown in the coordinate plane.

   ![Coordinate Plane with Triangles]

   a. Describe the positions of the two triangles on this coordinate plane. Do you think the two triangles are congruent?

   b. Place a mirror on the y-axis facing to the left. Describe what you see when you look at the triangle in the mirror.

   c. Place a mirror on the y-axis facing to the right. Describe what you see when you look at the triangle in the mirror.

Figures that are mirror images of each other are called reflections. A reflection is a transformation that “flips” a figure across a reflection line. A reflection line is a line that acts as a mirror so that corresponding points are the same distance from the mirror. In this coordinate plane, either triangle is a reflection of the other.

d. What do you think is the reflection line in the diagram shown?
e. Draw the reflection of each of the triangles across the x-axis.

2. Reflect parallelogram $ABCD$, using the y-axis as the reflection line, to form parallelogram $A'B'C'D'$.

a. Connect each vertex of the original parallelogram to the corresponding vertex of the image with line segments $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and $\overline{DD'}$.

b. Describe the relationship between the y-axis and each of the segments you drew.
c. List the ordered pairs for the vertices of parallelogram $ABCD$ and parallelogram $A'B'C'D'$.

**d.** What do you notice about the ordered pairs of the vertices of the original figure and its reflection across the $y$-axis?

3. Reflect parallelogram $ABCD$ across the $x$-axis by using the $x$-axis as a perpendicular bisector.

**a.** List the ordered pairs for the vertices of the original parallelogram and the reflected image.

---

You might want to create a table to organize your ordered pairs.
b. What do you notice about the ordered pairs of the vertices of the original figure and its reflection across the x-axis?

4. A triangle has vertices at \( A(-4, 3), B(1, 5), C(2, -2) \).
   a. If this triangle is reflected across the x-axis, what would the ordered pairs of the reflection's vertices be?
   
   b. If this triangle is reflected across the y-axis, what would the ordered pairs of the reflection's vertices be?

**Problem 2  Reflections Across Horizontal and Vertical Lines**

1. Reflect the triangle across the line \( x = -1 \).
2. Reflect the triangle across the line $y = 2$.

3. Recall that geometric figures are considered congruent when they are the same size and the same shape.
   
   a. Did reflecting the triangle on the coordinate plane change the size or shape of the figure?
   
   b. Is the image of the reflection of the triangle congruent to the original figure shown on the coordinate plane? Explain your reasoning.
1. Are all images, or new figures that result from a reflection, always congruent to the original figure? Explain.

2. Describe the line of reflection in each.
   a. 
   b. 

Be prepared to share your solutions and methods.
3.4 Translations, Rotations, and Reflections of Triangles

Learning Goals
In this lesson, you will:
- Translate triangles on a coordinate plane.
- Rotate triangles on a coordinate plane.
- Reflect triangles on a coordinate plane.

When you look at the night sky, you see bright stars and dim stars. But are the dimmer stars farther away from us or just less bright? Astronomers use a variety of methods to measure the universe, but at the end of the 1980s, they made vast improvements in the accuracy of these measurements.

In 1989, the Hipparcos satellite was launched by the European Space Agency. Among other advantages, this satellite was not affected by Earth's atmosphere and could view the entire “sky,” so it could provide more accurate measurements of distances. In 1997, the Hipparcos Catalogue was published, which contained high-precision distance information for more than 100,000 stars!
Problem 1 Translating Triangles on a Coordinate Plane

You have studied translations, rotations, and reflections of various geometric figures. In this lesson, you will explore, compare, and generalize the characteristics of triangles as you translate, rotate, and reflect them on a coordinate plane.

Consider the point \((x, y)\) located anywhere in the first quadrant of the coordinate plane.

### 1. Translate the point \((x, y)\) according to the descriptions in the table shown. Plot the point, and then record the coordinates of the translated points in terms of \(x\) and \(y\).

<table>
<thead>
<tr>
<th>Translation</th>
<th>Point ((x, y)) located in Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 units to the left</td>
<td></td>
</tr>
<tr>
<td>3 units down</td>
<td></td>
</tr>
<tr>
<td>3 units to the right</td>
<td></td>
</tr>
<tr>
<td>3 units up</td>
<td></td>
</tr>
</tbody>
</table>
2. Describe the translation in terms of $x$ and $y$ that would move any point $(x, y)$ into:
   a. Quadrant II
   b. Quadrant III
   c. Quadrant IV

3. Graph triangle $ABC$ by plotting the points $A(-3, 4), B(-6, 1),$ and $C(-4, 9)$.

   Use the table to record the coordinates of the vertices of each triangle.
   a. Translate triangle $ABC$ 5 units to the right to form triangle $A'B'C'$. List the coordinates of points $A', B',$ and $C'$. Then graph triangle $A'B'C'$.
   b. Translate triangle $ABC$ 8 units down to form triangle $A''B''C''$. List the coordinates of points $A'', B''$, and $C''$. Then graph triangle $A''B''C''$.

Can you translate a point from QI to QIII in one move? 

Triangle $ABC$ is located in Quadrant II. Do you think any of these translations will change the quadrant location of the triangle?
Let's consider the vertices of a different triangle and translations without graphing.

4. The vertices of triangle DEF are \( D(-7, 10) \), \( E(-5, 5) \), and \( F(-8, 1) \).

   a. If triangle DEF is translated to the right 12 units, what are the coordinates of the vertices of the image? Name the triangle.

   b. How did you determine the coordinates of the image without graphing the triangle?

   c. If triangle DEF is translated up 9 units, what are the coordinates of the vertices of the image? Name the triangle.

   d. How did you determine the coordinates of the image without graphing the triangle?

<table>
<thead>
<tr>
<th>Original Triangle</th>
<th>Triangle Translated 5 units to the Right</th>
<th>Triangle Translated 8 units Down</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \triangle ABC )</td>
<td>( \triangle A'B')C'</td>
<td>( \triangle A''B''C'' )</td>
</tr>
<tr>
<td>( A (-3, 4) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B (-6, 1) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C (-4, 9) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Problem 2  Rotating Triangles on a Coordinate Plane**

1. Graph the point \((x, y)\) anywhere in the first quadrant of the coordinate plane.

   Use the table to record the coordinates of each point.

   a. Using the origin \((0, 0)\) as the point of rotation, rotate point \((x, y)\) 90° counterclockwise about the origin and graph the rotated point on the coordinate plane. What are the new coordinates of the rotated point in terms of \(x\) and \(y\)?

   b. Using the origin \((0, 0)\) as the point of rotation, rotate point \((x, y)\) 180° counterclockwise about the origin and graph the rotated point on the coordinate plane. What are the new coordinates of the rotated point in terms of \(x\) and \(y\)?

   c. Using the origin \((0, 0)\) as the point of rotation, rotate point \((x, y)\) 270° counterclockwise about the origin and graph the rotated point on the coordinate plane. What are the new coordinates of the rotated point in terms of \(x\) and \(y\)?

   d. Using the origin \((0, 0)\) as the point of rotation, rotate point \((x, y)\) 360° counterclockwise about the origin and graph the rotated point on the coordinate plane. What are the new coordinates of the rotated point in terms of \(x\) and \(y\)?

<table>
<thead>
<tr>
<th>Original Point</th>
<th>Rotation About the Origin 90° Counterclockwise</th>
<th>Rotation About the Origin 180° Counterclockwise</th>
<th>Rotation About the Origin 270° Counterclockwise</th>
<th>Rotation About the Origin 360° Counterclockwise</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x, y))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Graph triangle $ABC$ by plotting the points $A(3, 4), B(6, 1),$ and $C(4, 9)$.

```
<table>
<thead>
<tr>
<th>Original Triangle</th>
<th>Rotation About the Origin 90° Counterclockwise</th>
<th>Rotation About the Origin 180° Counterclockwise</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle ABC$</td>
<td>$\triangle A'B'C'$</td>
<td>$\triangle A''B''C''$</td>
</tr>
<tr>
<td>$A (3, 4)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B (6, 1)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C (4, 9)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

Think about your answers from Question 1 as you rotate the triangle.
Let’s consider a different triangle and rotations without graphing.

3. The vertices of triangle DEF are $D(-7, 10)$, $E(-5, 5)$, and $F(-1, -8)$.
   a. If triangle DEF is rotated 90° counterclockwise, what are the coordinates of the vertices of the image? Name the rotated triangle.

   b. How did you determine the coordinates of the image without graphing the triangle?

   c. If triangle DEF is rotated 180° counterclockwise, what are the coordinates of the vertices of the image? Name the rotated triangle.

   d. How did you determine the coordinates of the image without graphing the triangle?
Problem 3  Reflecting Triangles on a Coordinate Plane

1. Graph the point \((x, y)\) anywhere in the first quadrant of the coordinate plane.

Use the table to record the coordinates of each point.

- **a.** Reflect and graph the point \((x, y)\) across the \(x\)-axis on the coordinate plane. What are the new coordinates of the reflected point in terms of \(x\) and \(y\)?
- **b.** Reflect and graph the point \((x, y)\) across the \(y\)-axis on the coordinate plane. What are the new coordinates of the reflected point in terms of \(x\) and \(y\)?:

<table>
<thead>
<tr>
<th>Original Point</th>
<th>Reflection Across the (x)-axis</th>
<th>Reflection Across the (y)-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x, y))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Graph triangle $ABC$ by plotting the points $A(3, 4)$, $B(6, 1)$, and $C(4, 9)$.

Use the table to record the coordinates of the vertices of each triangle.

<table>
<thead>
<tr>
<th>Original Triangle</th>
<th>Triangle Reflected Across the x-axis</th>
<th>Triangle Reflected Across the y-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle ABC$</td>
<td>$\triangle A'B'C'$</td>
<td>$\triangle A''B''C''$</td>
</tr>
<tr>
<td>$A$ (3, 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$ (6, 1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C$ (4, 9)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Do you see any patterns?

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Let's consider a different triangle and reflections without graphing.

3. The vertices of triangle $DEF$ are $D(-7, 10)$, $E(-5, 5)$, and $F(-1, -8)$.

a. If triangle $DEF$ is reflected across the $x$-axis, what are the coordinates of the vertices of the image? Name the triangle.

b. How did you determine the coordinates of the image without graphing the triangle?

c. If triangle $DEF$ is reflected across the $y$-axis, what are the coordinates of the vertices of the image? Name the triangle.

d. How did you determine the coordinates of the image without graphing the triangle?
Talk the Talk

1. The vertices of triangle PQR are P(4, 3), Q(−2, 2), and R(0, 0). Describe the translation used to form each triangle. Explain your reasoning.
   a. P'(0, 3), Q'(−6, 2), and R'(−4, 0)
   b. P'(4, 5.5), Q'(−2, 4.5), and R'(0, 2.5)

2. The vertices of triangle JME are J(1, 3), M(6, 5), and E(8, 1). Describe the rotation used to form each triangle. Explain your reasoning.
   a. J'(−3, 1), M'(−5, 6), and E'(−1, 8)
   b. J'(−1, −3), M'(−6, −5), and E'(−8, −1)
3. The vertices of triangle $NRT$ are $N(12, 4)$, $R(14, 1)$, and $T(20, 9)$. Describe the reflection used to form each triangle. Explain your reasoning.

a. $N'(-12, 4)$, $R'(-14, 1)$, and $T'(-20, 9)$

b. $N'(12, -4)$, $R'(14, -1)$, and $T'(20, -9)$

4. Are all the images that result from a translation, rotation, or reflection (always, sometimes, or never) congruent to the original figure?

Remember, congruence preserves size and shape.

Be prepared to share your solutions and methods.
Learning Goals
In this lesson, you will:

- Identify corresponding sides and corresponding angles of congruent triangles.
- Explore the relationship between corresponding sides of congruent triangles.
- Explore the relationship between corresponding angles of congruent triangles.
- Write statements of triangle congruence.
- Identify and use transformations to create new images.

Key Terms
- congruent line segments
- congruent angles
- corresponding sides
- corresponding angles

In mathematics, when a geometric figure is translated, reflected, or rotated, the size and shape of the figure doesn’t change. But in physics, things are a little different. An idea called length contraction in physics means that when an object is in motion, its length appears to be slightly less than it really is. The faster the object is moving, the smaller it appears. If an object is moving at the speed of light, it would be practically invisible!
Problem 1  Understanding Congruence

In the previous lesson, you determined that if a triangle was translated, rotated, or reflected, it resulted in creating an image that was the same size and the same shape as the original triangle; therefore, the image and the original triangle are said to be congruent triangles.

**Congruent line segments** are line segments that have the same length. Congruent triangles are triangles that are the same size and the same shape.

If the length of line segment $AB$ is equal to the length of line segment $DE$, the relationship can be expressed using symbols. These are a few examples.

- $AB = DE$ is read “the distance between $A$ and $B$ is equal to the distance between $D$ and $E$”
- $m\overline{AB} = m\overline{DE}$ is read “the measure of line segment $AB$ is equal to the measure of line segment $DE$.”

If the sides of two different triangles are equal in length, for example, the length of side $AB$ in triangle $ABC$ is equal to the length of side $DE$ in triangle $DEF$, these sides are said to be congruent. This relationship can be expressed using symbols.

- $\overline{AB} \cong \overline{DE}$ is read “line segment $AB$ is congruent to line segment $DE$.”

**Congruent angles** are angles that are equal in measure.

If the measure of angle $A$ is equal to the measure of angle $D$, the relationship can be expressed using symbols.

- $m\angle A = m\angle D$ is read “the measure of angle $A$ is equal to the measure of angle $D$.”

If the angles of two different triangles are equal in measure, for example, the measure of angle $A$ in triangle $ABC$ is equal to the measure of angle $D$ in triangle $DEF$, these angles are said to be congruent. This relationship can be expressed using symbols.

- $\angle A \cong \angle D$ is read “angle $A$ is congruent to angle $D$.”
Problem 2  Corresponding Sides of Congruent Triangles

Let’s explore the properties of congruent triangles.

1. Graph triangle ABC by plotting the points A(8, 10), B(1, 2), and C(8, 2).

   a. Describe triangle ABC.

   b. Use the coordinate plane to determine the lengths of sides AC and BC.

   c. Use the Pythagorean Theorem to determine the length of side AB.

2. Translate triangle ABC 10 units to the left to form triangle DEF.
   Graph triangle DEF and list the coordinates of points D, E, and F.
**Corresponding sides** are sides that have the same relative positions in geometric figures.

Triangle $ABC$ and triangle $DEF$ in Question 1 are the same size and the same shape. Each side in triangle $ABC$ matches or corresponds to a specific side in triangle $DEF$.

3. What would you predict to be true about the lengths of corresponding sides of congruent triangles?

4. Identify the pairs of corresponding sides of triangle $ABC$ and triangle $DEF$.
   a. Side $AC$ in triangle $ABC$ corresponds to what side in triangle $DEF$?
   b. Side $BC$ in triangle $ABC$ corresponds to what side in triangle $DEF$?
   c. Side $AB$ in triangle $ABC$ corresponds to what side in triangle $DEF$?

5. Determine the side lengths of triangle $DEF$.
   a. $m\overline{DF}$
   b. $m\overline{EF}$
   c. $m\overline{DE}$

6. Compare the lengths of the sides in triangle $ABC$ to the lengths of the corresponding sides in triangle $DEF$.
   a. How does the length of side $AC$ compare to the length of side $DF$?
   b. How does the length of side $BC$ compare to the length of side $EF$?
   c. How does the length of side $AB$ compare to the length of side $DE$?

7. In general, what can be said about the relationship between the corresponding sides of congruent triangles?
Problem 3  Corresponding Angles of Congruent Triangles

Use triangle ABC and triangle DEF in Problem 2 to answer each question.

1. Use a protractor to determine the measure of angles A, B, and C.

Triangle ABC and triangle DEF are the same size and the same shape. Each angle in triangle ABC matches, or corresponds, to a specific angle in triangle DEF. Corresponding angles are angles that have the same relative positions in geometric figures.

2. What would you predict to be true about the measures of corresponding angles of congruent triangles?

3. Identify the corresponding angles of triangle ABC and triangle DEF.
   a. Angle A in triangle ABC corresponds to what angle in triangle DEF?
   b. Angle B in triangle ABC corresponds to what angle in triangle DEF?
   c. Angle C in triangle ABC corresponds to what angle in triangle DEF?

4. Use a protractor to determine the measures of angles D, E, and F.
5. Compare the measures of the angles in triangle $ABC$ to the measures of the corresponding angles in triangle $DEF$.
   
a. How does the measure of angle $A$ compare to the measure of angle $D$?
   
b. How does the measure of angle $B$ compare to the measure of angle $E$?
   
c. How does the measure of angle $C$ compare to the measure of angle $F$?

6. In general, what can be said about the relationship between the corresponding angles of congruent triangles?

**Problem 4  Statements of Triangle Congruence**

1. Consider the congruence statement $\triangle JRB \cong \triangle MNS$.
   
a. Identify the congruent angles.  
b. Identify the congruent sides.

So, if you know that two angles are congruent then you also know their measures are equal.
2. Analyze the two triangles shown.

![Graph with points labeled P, M, K, W, T, C, and W']

a. Identify the transformation used to create triangle PMK.

b. Does the transformation used preserve the size and shape of the triangle?

c. Using the triangles shown, write a triangle congruence statement.

d. Using your congruence statement, identify the congruent angles.

e. Using your congruence statement, identify the congruent sides.
3. Analyze the two triangles shown.

![Graph showing two triangles, labeled ZQV and another set of coordinates.]

a. Identify the transformation used to create triangle ZQV.

b. Does the transformation used preserve the size and shape of the triangle?

c. Using the triangles shown, write a triangle congruence statement.

d. Using your congruence statement, identify the congruent angles.

e. Using your congruence statement, identify the congruent sides.
1. Given any triangle on a coordinate plane, how can you create a different triangle that you know will be congruent to the original triangle?

2. Describe the properties of congruent triangles.

Be prepared to share your solutions and methods.
Chapter 3 Summary

Key Terms
- transformation (3.1)
- translation (3.1)
- image (3.1)
- pre-image (3.1)
- rotation (3.2)
- angle of rotation (3.2)
- point of rotation (3.2)
- reflection (3.3)
- reflection line (3.3)
- congruent line segments (3.5)
- congruent angles (3.5)
- corresponding sides (3.5)
- corresponding angles (3.5)

3.1 Translating Geometric Figures
A translation is a transformation that “slides” each point of a figure the same distance and direction. Sliding a figure left or right is a horizontal translation and sliding it up or down is a vertical translation. The new figure created from a translation is called the image.

Example
$\triangle ABC$ with coordinates $A(-2, 2), B(0, 5),$ and $C(1, 1)$ is translated six units horizontally and $-4$ units vertically.

The coordinates of the image are $A'(4, -2), B'(6, 1),$ and $C'(7, -3)$.

Feel like you still don’t understand something new? That’s OK. Ask questions, practice more problems, and get involved. You will understand it better after you do!
A rotation is a transformation that turns a figure about a fixed point for a given angle and a given direction. The given angle is called the angle of rotation. The angle of rotation is the amount of rotation about a fixed point. The point around which the figure is rotated is called the point of rotation. Rotations can be either clockwise or counterclockwise.

**Example**

To rotate $\triangle XYZ$ $45^\circ$ clockwise around point $Z$, use a protractor to draw a $45^\circ$ angle as shown, with point $Z$ as the vertex. Next, rotate the figure clockwise around point $Z$ until the side corresponding to $YZ$ has been rotated $45^\circ$. The image is labeled as $\triangle X'Y'Z$. 
3.3 Reflecting Geometric Figures on the Coordinate Plane

A reflection is a transformation that “flips” a figure across a reflection line. A reflection line is a line that acts as a mirror such that corresponding points in the figure and its image are the same distance from the line.

When a figure is reflected across the $x$-axis, the $y$-values of the points on the image have the opposite sign of the $y$-values of the corresponding points on the original figure while the $x$-values remain the same. When a figure is reflected across the $y$-axis, the $x$-values of the points on the image have the opposite sign of the $x$-values of the corresponding points on the original figure while the $y$-values remain the same.

Example

A square with vertices $P(2, 1), Q(2, 8), R(5, 5),$ and $S(2, 2)$ is reflected across the $x$-axis.

To determine the vertices of the image, change the sign of the $y$-coordinates of the figure’s vertices to find the $y$-coordinates of the image’s vertices. The $x$-coordinates remain the same. The vertices of the image are $P'(2, -1), Q'(2, -8), R'(5, -5),$ and $S'(2, -2).$
Translating Triangles in the Coordinate Plane

To translate a triangle in the coordinate plane means to move or “slide” the triangle to a new location without rotating it.

Example

Triangle $ABC$ has been translated 10 units to the left and 2 units down to create triangle $A'B'C'$.

The coordinates of triangle $ABC$ are $A(2, 8), B(7, 5),$ and $C(2, 5)$.

The coordinates of triangle $A'B'C'$ are $A'(-8, 6), B'(-3, 3),$ and $C'(-8, 3)$.
3.4 Rotating Triangles in the Coordinate Plane

To rotate a triangle in the coordinate plane means to “turn” the triangle either clockwise or counterclockwise about a fixed point, which is usually the origin. To determine the new coordinates of a point after a rotation, refer to the following table.

<table>
<thead>
<tr>
<th>Original Point</th>
<th>Rotation About the Origin 90° Counterclockwise</th>
<th>Rotation About the Origin 180° Counterclockwise</th>
<th>Rotation About the Origin 270° Counterclockwise</th>
<th>Rotation About the Origin 360° Counterclockwise</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x, y)</td>
<td>(−y, x)</td>
<td>(−x, −y)</td>
<td>(y, −x)</td>
<td>(x, y)</td>
</tr>
</tbody>
</table>

Example

Triangle \(ABC\) has been rotated 180° counterclockwise about the origin to create triangle \(A'B'C'\).

The coordinates of triangle \(ABC\) are \(A(2, 8), B(7, 5),\) and \(C(2, 5)\).

The coordinates of triangle \(A'B'C'\) are \(A'(-2, -8), B'(-7, -5),\) and \(C'(-2, -5)\).
3.4 Reflecting Triangles on a Coordinate Plane

To reflect a triangle on a coordinate plane means to “mirror” the triangle across a line of reflection to create a new triangle. Each point in the new triangle will be the same distance from the line of reflection as the corresponding point in the original triangle. To determine the coordinates of a point after a reflection across the x-axis, change the sign of the y-coordinate in the original point. To determine the coordinates of a point after a reflection across the y-axis, change the sign of the x-coordinate in the original point.

Example

Triangle $ABC$ has been reflected across the x-axis to create triangle $A'B'C'$.

The coordinates of triangle $ABC$ are $A(2, 8)$, $B(7, 5)$, and $C(2, 5)$.

The coordinates of triangle $A'B'C'$ are $A'(2, -8)$, $B'(7, -5)$, and $C'(2, -5)$. 
3.5 Identifying Corresponding Sides and Angles of Congruent Triangles

Congruent figures are figures that are the same size and the same shape. Congruent triangles are triangles that are the same size and the same shape. Congruent line segments are line segments that are equal in length. Congruent angles are angles that are equal in measure. In congruent figures, the corresponding angles are congruent and the corresponding sides are congruent.

Example

Triangle $DEF$ has been reflected across the $y$-axis to create triangle $PQR$.

- Line segment $DE$ corresponds to line segment $PQ$, which means $DE \cong PQ$.
- Line segment $EF$ corresponds to line segment $QR$, which means $EF \cong QR$.
- Line segment $DF$ corresponds to line segment $PR$, which means $DF \cong PR$.
- Angle $D$ corresponds to angle $P$, which means $\angle D \cong \angle P$.
- Angle $E$ corresponds to angle $Q$, which means $\angle E \cong \angle Q$.
- Angle $F$ corresponds to angle $R$, which means $\angle F \cong \angle R$.

The congruence statement for these two congruent triangles is $\triangle DEF \cong \triangle PQR$. 

![Diagram of congruent triangles](image-url)