Latent Heat Calculations

What heat quantities are needed to change a substance from a liquid to a gas? How much energy is needed to melt a solid to a liquid? Is it the same for each substance?

Let’s look at a graph for boiling 50 grams of water from about room temperature, 20 °C:

You’ll notice the two parts of the graph we discussed last section. The first part is the 50 grams of water heating from 20 °C up to its boiling temperature, 100 °C. The second part is boiling the water at 100 °C.

How much heat is needed? The first part is easy, we have done this before:

\[ Q = mc (T_f - T_i) \]

\[ Q = (50 \text{ grams}) (1 \text{ cal/g °C})(100 \text{ °C} - 20 \text{ °C}) \]

\[ Q = 4000 \text{ calories} \]
The second part of the graph is more difficult, though. If we try to use our equation:

\[ Q = mc(T_f - T_i) \]

\[ Q = (50 \text{ grams}) (1 \text{ cal/g } ^\circ\text{C})(100 ^\circ\text{C} - 100 ^\circ\text{C}) \]

\[ Q = 0 \text{ calories} \]

Notice that in part 2, there is no temperature change. Mathematically, then, that results in a heat quantity of zero. Does that mean that it take NO energy to boil water? If we turn off the heat, the temperature drops so no, this cannot be so we must need a different equation.

Why do we need a new equation for these phase changes? Remember back what temperature really measures. Temperature, we recall, is an approximate measure of the average kinetic energy of the particles in a sample. Look at part 2 of the graph, the temperature is not changing but when we boil water, we continue to put energy into it to turn it from a liquid to a gas. Adding this extra energy, however, does NOT increase the temperature and therefore does not increase the kinetic energy of the particles in the sample. Where is this energy going, then?

To answer it, you must think about what is happening. During the flat section, part 2, we are in the process of turning the liquid water into gaseous water. The temperature cannot increase past 100 °C until all the water is boiled away. Since we are not changing the kinetic energy of the sample (because of the lack of temperature change), we must be changing the potential energy of the sample. Think of it this way, by putting more energy into the system we are giving it the potential to reach higher temperatures.
To account for this difference, our equation for phase changes must be modified. The kinetic energy equation:

\[ Q = mc(T_f - T_i) \]

Involves heat, mass, specific heat capacity and the temperature change. For phase changes, though, the temperature doesn’t change so we remove the \( T_f \) and the \( T_i \). We also need to modify \( c \), the specific heat capacity, to a factor which describes the energy of phase changes, Latent Heat.

Latent heat was discussed in the last section and we determined there are two values we will deal with:

- Latent Heat of Fusion: energy needed to change from a solid to a liquid or back
- Latent Heat of Vaporization: energy needed to change from a liquid to a gas or back

We will now give each of these factors a symbol:

- Latent Heat of Fusion: \( L_f \)
- Latent Heat of Vaporization: \( L_v \)

Getting rid of the temperature change and substituting \( L_f \) or \( L_v \) into the equation we get:

\[ Q = mL_f \]
\[ Q = mL_v \]

Use the first equation for any change involving freezing or melting. Use the second equation for any change involving boiling or condensing.

Different substances have different values for the energy needed to melt or boil it. Here is a table of values:

<table>
<thead>
<tr>
<th>Substance</th>
<th>Heat of Fusion, ( L_f ) (cal/gram)</th>
<th>Heat of Vaporization, ( L_v ) (cal/gram)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>80</td>
<td>540</td>
</tr>
<tr>
<td>Sulfur</td>
<td>9.1</td>
<td>78</td>
</tr>
<tr>
<td>Gold</td>
<td>15.4</td>
<td>378</td>
</tr>
<tr>
<td>Copper</td>
<td>32.1</td>
<td>1213</td>
</tr>
<tr>
<td>Alcohol</td>
<td>24.9</td>
<td>204</td>
</tr>
</tbody>
</table>
Example 1:
A 25 gram ice cube is placed on the counter. After a while it melts. How much energy was required to melt it?

\[ Q = mL_f \]

\[ m = 25 \text{ gram} \]
\[ L_f = 80 \text{ cal/gram} \text{ (from the table on the previous page)} \]

\[ Q = (25 \text{ g})(80 \text{ cal/g}) = 2000 \text{ calories} \]

Example 2:
A 25 gram block of gold is heated to its melting point. More energy is put in and the gold melts. How much energy was required to melt it?

\[ Q = mL_f \]

\[ m = 25 \text{ gram} \]
\[ L_f = 15.4 \text{ cal/gram} \text{ (from the table on the previous page)} \]

\[ Q = (25 \text{ g})(15.4 \text{ cal/g}) = 385 \text{ calories} \]

Example 3:
A beaker of water is heated to boiling. The water has a mass of 400 grams. How much energy will be needed to boil it?

\[ Q = mL_v \]

\[ m = 400 \text{ gram} \]
\[ L_v = 540 \text{ cal/gram} \text{ (from the table on the previous page)} \]

\[ Q = (400 \text{ g})(540 \text{ cal/g}) = 216,000 \text{ calories} \]

Example 4:
A 75 gram sample of alcohol is heated until it boils. How many calories are needed to boil it?

\[ Q = mL_v \]

\[ m = 75 \text{ gram} \]
\[ L_v = 204 \text{ cal/gram} \text{ (from the table on the previous page)} \]

\[ Q = (75 \text{ g})(204 \text{ cal/g}) = 15,300 \text{ calories} \]
Example 5:
800 calories were needed to melt a substance that had a mass of 25 grams. What is the Heat of Fusion of the substance?

\[ Q = mL_f \]
\[ Q = 800 \text{ calories} \]
\[ m = 25 \text{ gram} \]
\[ L_f = x \]

\[ 800 \text{ cal} = (25 \text{ g})(x) \]
\[ X = 32 \text{ cal/g} \]

Example 6:
680,000 calories were needed to boil 1250 grams of a liquid at its boiling point. What is the Heat of Vaporization of the liquid?

\[ Q = mL_v \]
\[ Q = 680,000 \text{ calories} \]
\[ m = 1250 \text{ gram} \]
\[ L_v = x \]

\[ 680,000 \text{ cal} = (1250 \text{ g})(x) \]
\[ X = 544 \text{ cal/g} \]

Example 7:
How many grams of sulfur could be melted if 750 calories were used to heat it at its melting point?

\[ Q = mL_v \]
\[ Q = 750 \text{ calories} \]
\[ m = x \]
\[ L_f = 9.1 \text{ cal/gram} \text{ (from the table on the previous page)} \]

\[ 750 \text{ calories} (x)(9.1 \text{ cal/g}) \]
\[ X = 82.4 \text{ grams} \]
Questions

1. Why can we not use the equation $Q = mc\Delta T$ for phase changes of a substance; why won’t it work?
2. What is the Latent Heat of Fusion of a substance?
3. What is the Latent Heat of Vaporization of a substance?
4. How many calories are needed to melt 30 grams of copper?
5. How much heat is needed to melt 85 grams of gold?
6. How much heat is needed to boil 63 grams of alcohol?
7. How many calories are needed to boil 100 grams of water?
8. What is the Latent Heat of Fusion of a substance if 1200 calories melts 40 grams of it?
9. What is the Latent Heat of Vaporization of a substance if 50 calories boiled 3 grams of it?
10. How many grams of copper could be melted if 700 calories are applied to it?
11. How many grams of water could be boiled if 8000 calories are applied to it?